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HEATING OR COOLING A STABLE COMPOSITIONAL GRADIENT FROM THE SIDE

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Abstract—When fluid with a vertical compositional gradient is heated from the side, the resultant motion consists of a series of layers which propagate away from the heated wall. The layers are due to the fact that fluid parcels heated at the wall rise in a decreasing ambient density gradient which limits their ascent. The size of these layers as a function of the imposed Rayleigh number has been previously determined for a vertical salinity gradient. The present work examines the effect of changes in Prandtl and Lewis numbers by evaluating the layer sizes for the cases of heating a sucrose gradient or a glycerine gradient and cooling and crystallizing a sodium carbonate gradient. In all cases we find that the representation determined for the Prandtl and Lewis numbers appropriate for salt holds for the other cases.

NOMENCLATURE

c	constant of proportionality between h and η
h	thickness of the layer
Le	Lewis number
Pr	Prandtl number
S	salinity or composition
T	temperature.

Greek symbols

ρ	density
$d\rho/dz$	density gradient
η	thickness scale.

Subscripts

w	wall value
∞	far-field value.

1. INTRODUCTION

DOUBLE-diffusive convection describes the form of motion that occurs in fluids with two or more components of different molecular diffusivities which contribute in an opposing sense to the vertical density gradient. The subject is a relatively new area of research, and it is also a rapidly growing one, mainly due to its application to many different disciplines. Thus, engineers, fluid dynamicists, physicists, crystal growers, metallurgists, geologists and other scientists are all finding that double-diffusive convection impinges on parts of their subject [1]. In each area the appropriate components are different, but it is the overall general concepts which unite the various applications. For example, the components of concern to the solar pond engineer are either heat and salt or heat and some other solute [2]. The chemical engineer, on the other hand, may be interested in solutions with two (or more) different solutes [3]. Many of the foundations of the subject are described by Turner [4, 5] and a historical review is presented by Huppert and Turner [6].

A hallmark of double-diffusive convection is the transformation of a smooth vertical concentration gradient into a series of layers separated by very thin interfaces across which exist relatively large variations in concentrations. One way this can be achieved is by heating, or cooling, a smooth vertical compositional gradient from a side wall. This was first done in a controlled, quantitative manner by Thorpe *et al.* [7], who confined the fluid to a thin vertical slot. Then both the horizontal and vertical length scales of the motion are set by the width of the slot. Further experiments by Chen *et al.* [8] with a salinity gradient in a wide container indicated that beyond a critical Rayleigh number a series of layers extended from the heated side wall. The motion is driven by the rise of the warmed fluid parcels near the wall. Because the diffusion coefficient of salt is so very much less than that of heat, the parcels retain almost all their salt as they rise in a continually decreasing ambient density field. Their buoyancy deficiency also continually decreases and a level is reached beyond which no further vertical motion is possible; the parcel then turns horizontally into the interior. A series of horizontally oriented layers is thereby set up, as depicted in Fig. 1, taken from one of our experiments. This description of the forces which initiate the motion suggests that the layer thickness, h, will be directly proportional to

$$\eta = [\rho(T_\infty, S_\infty) - \rho(T_w, S_\infty)] \Big/ \frac{d\rho}{dz}, \tag{1}$$

where $\rho(T, S)$ is the density as a function of temperature and salinity, ∞ and w represent values in the far field and at the wall, respectively, and $d\rho/dz$ is the vertical density gradient due to salt. The constant of proportionality, c, between h and η could be a function of: the Rayleigh number

$$Ra = (g/\rho_0 \kappa_T \nu)[\rho(T_\infty, S_\infty) - \rho(T_w, S_\infty)]^4 (d\rho/dz)^{-3}; \tag{2}$$

the Prandtl number, Pr; and Le, the Lewis number, or

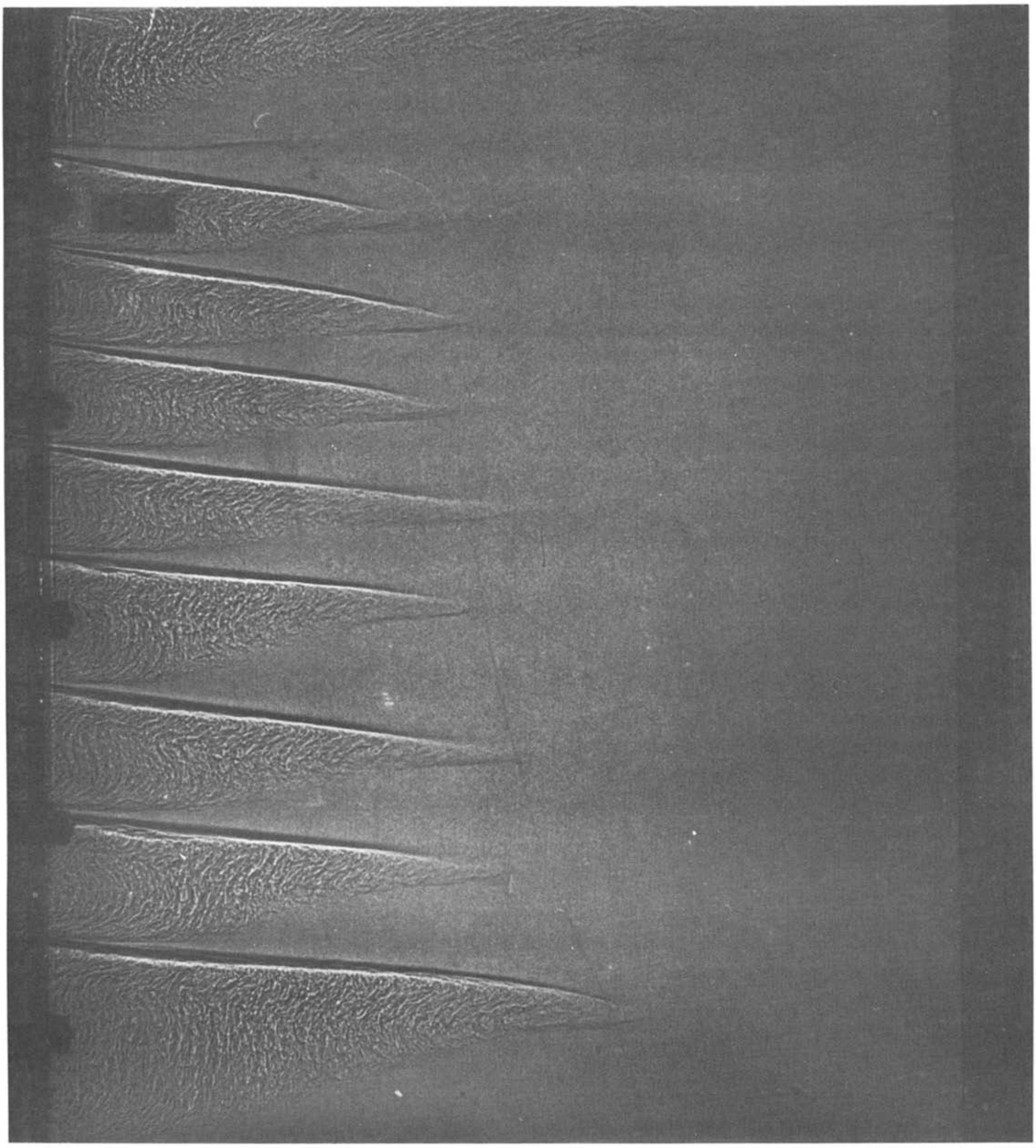

FIG. 1. A shadowgraph of a glycerine gradient which has been heated from the side for 7 min. Top: 0% glycerine, density 1.000 g cm^{-3}. Bottom: 22% glycerine, density 1.054 g cm^{-3}. Ambient temperature 19.0°C, wall temperature 41.5°C. Total layer depth 28.4 cm. Note the downward trend in the interfaces as they extend from the heated wall and decrease their temperature.

ratio of the molecular diffusivity of heat to that of the solute.

In order to determine the value of c as a function of Ra for heat and salt, for which $Pr = 7$ and $Le = 100$, Huppert and Turner [9] carried out a series of experiments with a variety of different vertical density gradients and horizontal temperature contrasts. They found that beyond a Rayleigh number of about 10^5, c was essentially independent of the Rayleigh number and given by

$$c = 0.62 \pm 0.05. \tag{3}$$

The aim of this communication is to discuss the value of c for different values of the Prandtl and Lewis numbers. We first describe, in Section 2, a series of experiments using sucrose as the solute, for which $Pr = 7$ and $Le = 300$. We next describe, in Section 3, a series of experiments using glycerine. By varying the amounts of glycerine the Prandtl number at 20°C can take values between 7 and 1.2×10^4 while the Lewis number takes values between 300 and 8000 [10]. Finally, in Section 4, we describe the effects of cooling and crystallizing a gradient of sodium carbonate, for which $Pr = 7$ and $Le = 200$. In all cases c is essentially constant and takes a value of around 0.6.

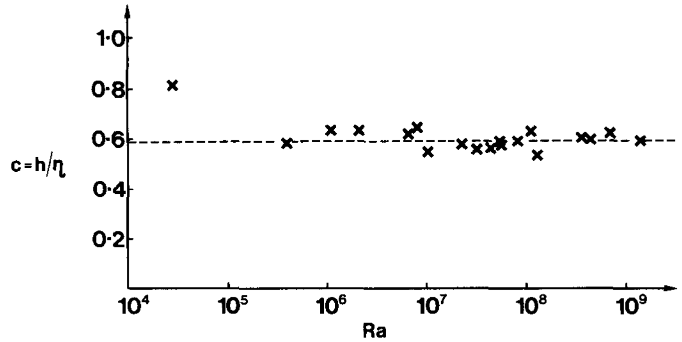

FIG. 2. The nondimensional layer thickness, $c = h/\eta$, as a function of Rayleigh number, Ra, for heating a sucrose gradient from the side. The dashed line represents the mean value 0.59.

2. HEATING A SUCROSE GRADIENT

All the experiments were carried out using a Perspex container, 50 × 20 × 40 cm high. A thin-walled aluminium box 14 × 20 × 40 cm high was placed against one end of the container. Hot water could be poured into this box and its temperature maintained by use of a thermoboy. The working space was filled to a nominal depth of 38 cm with fluid at room temperature, the vertical density gradient being produced by the standard double-bucket procedure [11]. A few experiments with salt were first employed to test the apparatus and successfully confirmed the earlier results of Huppert and Turner [9]. Since the experimental technique is identical to that reported there, we shall be rather brief in our description.

Nineteen experiments were carried out. The solutions all had fresh water at the surface and a linear density gradient which increased downwards to between 1.03 and 1.08 g cm^{-3} at the base. The water in the aluminium box was maintained at between 5 and 40°C above the initial temperature of the working fluid. The density of the sucrose solution as a function of temperature and of amount of sucrose was taken from standard tables [12].

The results of the various experiments are shown in Fig. 2. The value of c for $Ra > 10^5$ is seen to be essentially independent of the Rayleigh number and given by

$$c = 0.59 \pm 0.03. \quad (4)$$

3. HEATING A GLYCERINE GRADIENT

The use of sucrose rather than salt allows the Lewis number to be increased by a factor of 3. Use of glycerine solutions allows both the Prandtl number to be varied by a factor of about 2000 and the Lewis number by over a factor of 20. The disadvantage of using glycerine solutions is only that, because the viscosity can be such a strong function of concentration [10, 12], the viscosity, and hence local value of the Rayleigh number, can vary significantly. Since, however, it was suspected, and found, that the value of c is independent of the Rayleigh number, this is not a significant disadvantage.

A total of 25 experiments were conducted in a Perspex box 8.5 × 50 × 30 cm high. In the middle of the box there was a thin copper partition which separated the glycerine gradient from the heated water. For 13 of the experiments the solution was pure water at the surface and the density at the base varied between 1.02 and 1.10 g cm^{-3}. These experiments were filled using the standard double-bucket method [11]. A further 12 experiments were conducted for which the glycerine concentration at the surface exceeded 50% and increased with depth. The viscosity of these solutions was quite large and in order to fill the container the extension of the double-bucket method for viscous fluids described by Huppert *et al.* [13] was used.

The results of the various experiments are presented in Fig. 3. The horizontal lines centred on each experimental point reflect the variation in Rayleigh

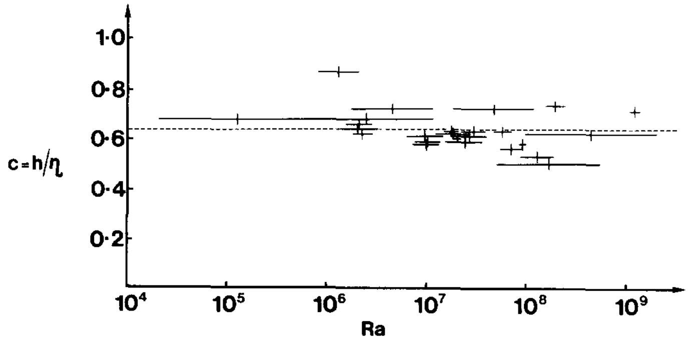

FIG. 3. The nondimensional layer thickness, $c = h/\eta$, as a function of Rayleigh number, Ra, for heating a glycerine gradient from the side. The horizontal lines about each data point indicate the range of Ra due to the variation of viscosity with height. The dashed line represents the mean value 0.64.

number with height. The right-hand end of each line represents equation (2) with values at the bottom of the tank and the left-hand end represents equation (2) with values at the surface. The experimental points show no systematic dependence on Rayleigh number and can be described by

$$c = 0.64 \pm 0.08. \qquad (5)$$

4. COOLING AND CRYSTALLIZING A Na_2CO_3 GRADIENT

Recently, research in double-diffusive convection has been extended to include effects of crystallization. This is of direct relevance to crystal growers, metallurgists, and geologists [1, 14]. The major new fluid mechanical phenomenon which occurs is that upon crystallization, liquid is released which is of a different density—generally lighter—than its surroundings. Thus crystallization by itself leads to convective motions. Application of this general idea to the particular configuration considered in this paper leads one to ask: what is the form, and size, of double-diffusive layers produced by cooling and crystallizing a compositional gradient from a side wall?

To answer this question we carried out a series of nine experiments using gradients of sodium carbonate (Na_2CO_3). We chose Na_2CO_3 because its solubility in water is large, its saturation concentration is a strong function of temperature, and because accurate data on its density as a function of temperature and concentration are readily available [15]. A phase diagram depicting this last property makes up Fig. 4. The experiments were carried out in the same Perspex container used for the sucrose experiments, only this time the water in the aluminium box was maintained at a fixed temperature in the range 3–12°C (room temperature ≈17°C), by use of a cooler. In all the experiments the concentration was, at room temperature, close to saturation at the base and decreased with height. In some experiments the concentration decrease was sufficiently large that the saturation temperature at the surface was below that imposed at the cold wall, while in others the surface saturation temperature was above the imposed temperature.

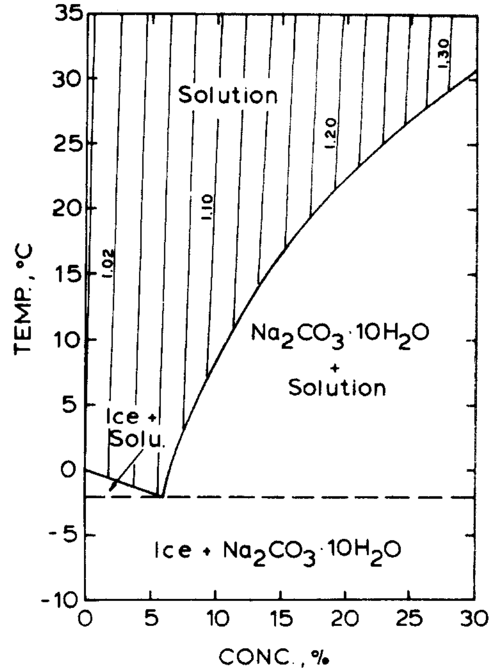

FIG. 4. The phase diagram and density of aqueous sodium carbonate. [Taken from C. F. Chen and J. S. Turner, Crystallization in a double-diffusive system, *J. Geophys. Res.* **85**, 2573–2593 (1980), with permission.]

The general behaviour in all the experiments was the same. Double-diffusive intrusions formed within a matter of minutes of pouring in the cold water and the fronts began to propagate away from the cooled wall, with velocities of the order of 1 cm min^{-1}. Evaluation of c by comparing the measured height of the layers with η given by equation (1) indicated again that it was constant, as presented in Fig. 5. The results of the nine experiments show that

$$c = 0.61 \pm 0.07. \qquad (6)$$

Observations of the evolution of the double-diffusive interfaces at later times revealed two effects. First, the growing crystal block on the cold wall partially insulated the lower portions of the fluid. This caused the interfaces to become indistinct, as indicated in Figs. 6 and 7, because the weakened convection is no longer able to keep them sharp against the action of diffusion. Second, there was a systematic decrease in the number of interfaces, due to either two interfaces merging, or to

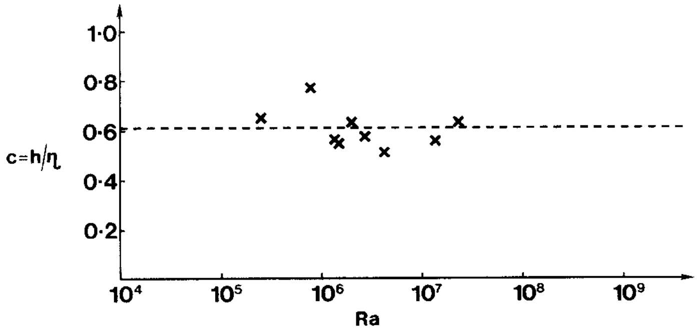

FIG. 5. The nondimensional layer thickness, $c = h/\eta$, as a function of Rayleigh number, Ra, for cooling and crystallizing a Na_2CO_3 gradient from the side. The dashed line represents the mean value 0.61.

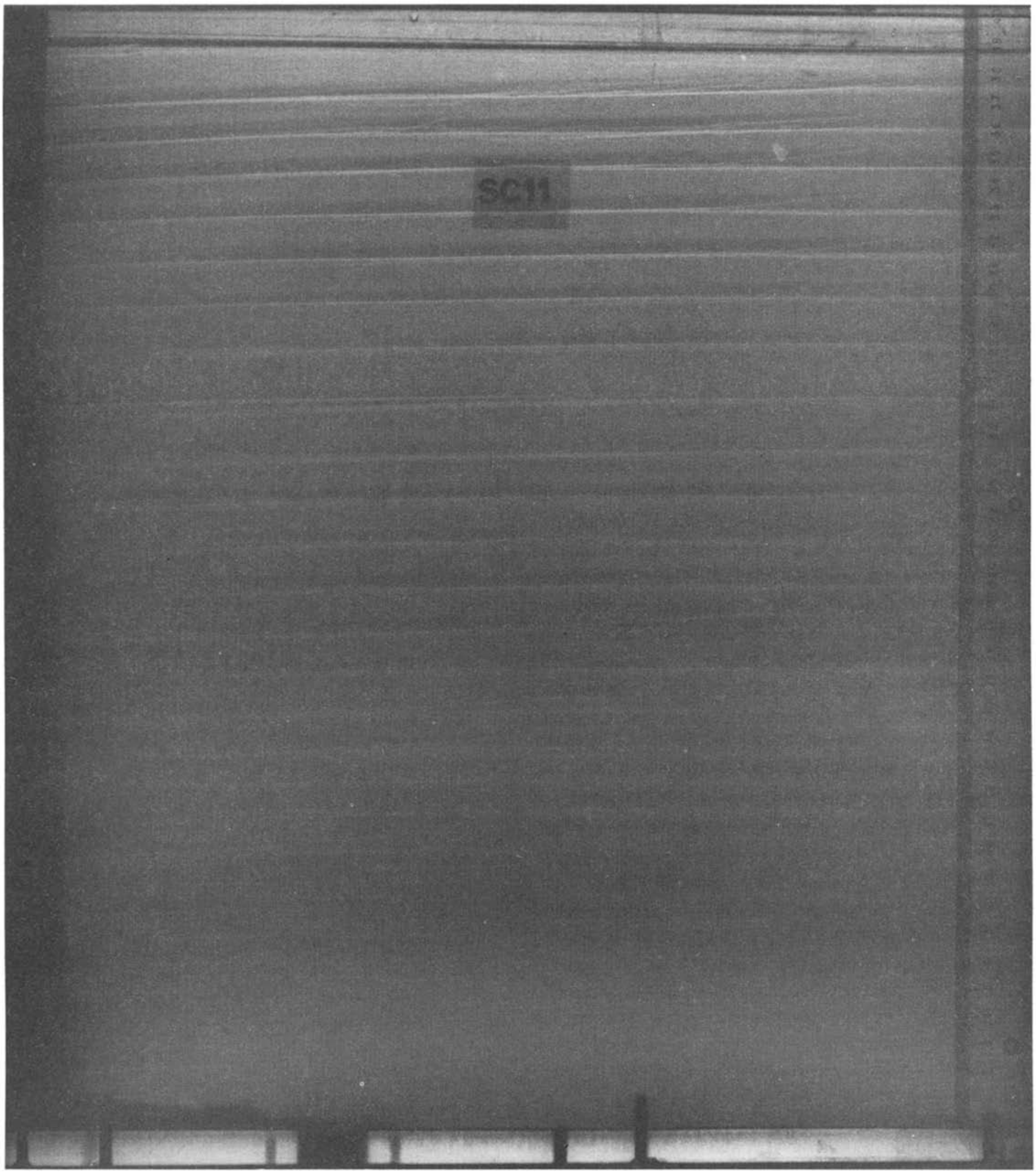

FIG. 6. A shadowgraph of a Na_2CO_3 gradient cooled from the side for 75 min. Top: 9.2 wt% Na_2CO_3, density 1.094 g cm^{-3}, saturation temperature 7°C. Bottom: 17.3 wt% Na_2CO_3, density 1.182 g cm^{-3}, saturation temperature 19°C. Ambient temperature 17.7°C, wall temperature 3.5°C. Total layer depth 38.8 cm. Note that in the lower 13 cm the crystal growth was sufficiently rapid to noticeably decrease the layer thickness, reflecting the insulating effect of the crystals.

one slowly weakening until it disappeared. As indicated in Fig. 7, this process occurred at all heights in the tank, which suggests that it is independent of the crystallization and release of light fluid. This conclusion was confirmed by heating a salinity gradient from the side and monitoring the evolution of the layers with time. The double-diffusive interfaces formed rapidly (and gave a value of c of 0.58). Subsequently, they weakened, merged or disappeared. This indicates that the formula $c = h/\eta = 0.6$ for layer thickness is valid only while the double-diffusive intrusions do not affect the far-field conditions. Once the intrusions have reached the opposing side wall of the tank, for example, the decrease in the horizontal temperature difference reduces the convection and allows diffusion to weaken and even destroy the initially sharp interfaces. In summary, when cooling from the side, the convection is weakened either by increasing T_w or by decreasing T_∞.

At the same time as the layers were developing in the Na_2CO_3 experiments, crystals of $Na_2CO_3.10H_2O$ were nucleating on the cooled wall. At first they formed in isolated clumps near the base of the tank, because the supersaturation was greatest there and thus favoured both a more rapid nucleation and a larger crystal

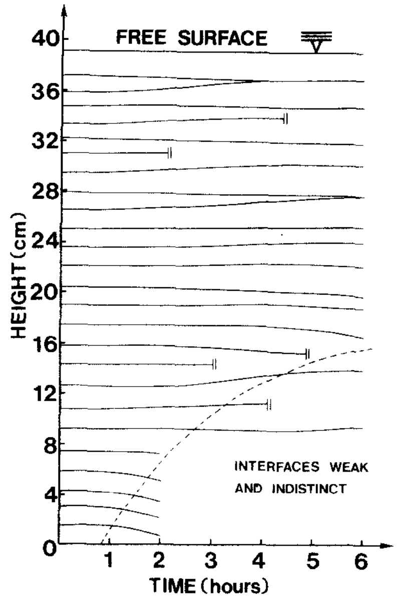

FIG. 7. The interface position as a function of time for an experiment cooling a Na_2CO_3 gradient. Top: 9.2 wt% Na_2CO_3, density 1.094 g cm^{-3}, saturation temperature 7°C. Bottom: 15.6 wt% Na_2CO_3, density 1.163 g cm^{-3}, saturation temperature 17°C. Ambient temperature 17.5°C, wall temperature 9.0°C. Total layer depth 39.2 cm. The dashed line indicates the height of the crystal block on the cooled wall as a function of time.

growth rate. Most of the early growth occurred as dendritic crystals radiating away from the initial nucleation sites. As time proceeded, the distribution of crystals across the cold wall became more uniform and extended to greater heights. Eventually, the dendritic growth ceased and a crystal block formed which was thickest at the base and fairly uniform across the cold wall. This indicates that the crystallization is a local effect and is unaffected by differing conditions in either horizontal or vertical directions.

Throughout the crystallization, fluid depleted in Na_2CO_3 rose above the crystals in the form of buoyant thermals. These thermals turbulently entrained ambient fluid until they reached a height at which they were neutrally buoyant. This height is unrelated to the layer thickness, but in our experiments was very much larger than it. Further details on the motion of the depleted fluid and the variation of the ambient density gradient with time is documented in the Prize Essay by Kerr [16].

The major point with which to conclude this section is that the production and rise of depleted fluid due to the crystallization does not affect the formation of layers or their size. This is because they respond to the horizontal temperature contrast, between wall and far field, and are not influenced by the flow of depleted fluid in a thin boundary layer near the wall. A similar result occurs when a vertical ice wall melts in a salinity gradient [9]. The upward flow of melted water close to the ice does not affect either the formation or the size of the layers.

5. CONCLUSIONS

Our work has presented the results of a series of experiments using fluids with different physical properties. As with all experimental investigations, one can not be confident about the possible results for parameter values different from those investigated. However, it appears that, at least for large Prandtl and Lewis numbers, heating, cooling or crystallizing a compositional gradient from the side causes an initially smooth gradient to be transformed to a series of layers. The initial size of the layers is well represented by $h = 0.6\eta$. If the ambient conditions change—for example, if the layers extend throughout the fluid—the interfaces separating the layers may weaken and the layer scales change.

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CHAUFFAGE OU REFROIDISSEMENT PAR UNE PAROI D'UN FLUIDE AVEC GRADIENT DE COMPOSITION

Résumé—Quand un fluide avec gradient vertical de composition est chauffé latéralement, le mouvement qui en résulte consiste en une série de couches qui se déplacent en s'écartant de la paroi. Les couches sont dues au fait que les particules fluides chauffées sur la paroi s'élèvent dans un gradient de densité décroissante lequel limite l'ascension. La taille de ces couches en fonction du nombre de Rayleigh imposé a été précédemment, déterminée pour un gradient de salinité. Cette étude considère l'effet du changement des nombres de Prandtl et de Lewis en évaluant les dimensions des couches pour les cas de chauffage avec un gradient de sucrose ou un gradient de glycérine, et le refroidissement et la cristallisation pour un gradient de carbonate de sodium. Dans tous les cas, on trouve que la représentation déterminée pour des nombres de Prandtl et de Lewis appropriés pour le sel est valable pour les autres cas.

HEIZEN ODER KÜHLEN EINES FLUIDS MIT EINEM STABILEN DICHTEGRADIENTEN

Zusammenfassung—Wenn ein Fluid mit einem vertikalen Dichtegradienten von der Seite beheizt wird, so besteht die resultierende Bewegung in einer Reihe von Schichten, die sich von der beheizten Wand weg bewegen. Die Schichten werden durch Fluidballen verursacht, die an der Wand aufgeheizt werden und in eine Umgebung mit abnehmender Dichte des umgebenden Fluids aufsteigen, was ihren Aufstieg begrenzt. Die Größe dieser Schichten als Funktion der Rayleigh-Zahl wurde schon früher an vertikalen Salzgradienten erforscht. Diese Arbeit untersucht den Einfluß der Änderungen der Prandtl- und der Lewis-Zahl durch Auswertung der Schichtdicken für den Fall der Beheizung eines Zucker-oder Glyzeringradienten und der Kühlung und Kristallisation eines Sodagradienten. In allen Fällen fanden wir, daß die für Salze passende Darstellung der Prandtl- und der Lewis-Zahl sich auch für die anderen Fälle eignet.

ОДНОСТОРОННИЙ НАГРЕВ ИЛИ ОХЛАЖДЕНИЕ ЖИДКОСТИ С УСТОЙЧИВЫМ ГРАДИЕНТОМ КОНЦЕНТРАЦИИ КОМПОНЕНТОВ

Аннотация—При одностороннем (боковом) нагреве жидкости с вертикальным градиентом концентрации компонентов возникает послойное течение от нагреваемой стенки. Слои образуются в результате того, что объемы жидкости у нагретой стенки поднимаются против падающего градиента плотности. Ранее при вертикальном градиенте солености размер слоев определялся как функция числа Рэлея. В данной работе по размерам слоев изучается влияние чисел Прандтля и Льюиса на величину градиента плотности при нагревании сахарозы или глицерина и охлаждении и кристаллизации карбоната натрия. Как выяснилось, во всех случаях полученное для соли выражение для чисел Прандтля и Льюиса остается справедливым и для других веществ.