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Stokes drift in coral reefs with depth-varying permeability

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In his famous paper of 1847 (Stokes GG. 1847 On the theory of oscillatory waves. *Trans. Camb. Phil. Soc.* **8**, 441–455), Stokes introduced the drift effect of particles in a fluid that is undergoing wave motion. This effect, now known as Stokes drift, is the result of differences between the Lagrangian and Eulerian velocities of the fluid element and has been well-studied, both in the laboratory and as a mechanism of mass transport in the oceans. On a smaller scale, it is of vital importance to the hydrodynamics of coral reefs to understand drift effects arising from waves on the ocean surface, transporting nutrients and oxygen to the complex ecosystems within. A new model is proposed for a class of coral reefs in shallow seas, which have a permeable layer of depth-varying permeability. We then note that the behaviour of the waves above the reef is only affected by the permeability at the top of the porous layer, and not its properties within, which only affect flow inside the porous layer. This model is then used to describe two situations found in coral reefs; namely, algal layers overlying the reef itself and reef layers whose permeability decreases with depth.

This article is part of the theme issue 'Stokes at 200 (part 2)'.

1. Introduction

Stokes [1] was the first to consider the difference between the Lagrangian and Eulerian velocities of fluid 'particles' undergoing periodic wave motion due to surface gravity waves, and remarked that this difference results in a net drift effect in the direction of wave propagation. This result was derived by Stokes for the case of an inviscid fluid described by a velocity potential ϕ, satisfying Laplace's equation $\nabla^2\phi = 0$. From the imposition of boundary conditions at the free surface and the bottom of the flow, a dispersion relationship for the waves is calculated, and then drifts can be evaluated by comparing the two velocity fields.

Following the approach of Phillips [2], if a fluid particle starts at position $\boldsymbol{x_0}$ and has velocity $\boldsymbol{u}_L(\boldsymbol{x_0}, t)$ in the Lagrangian description, its position at time t is given by

$$\boldsymbol{x}\left(\boldsymbol{x_0}, t\right) = \boldsymbol{x_0} + \int_0^t \boldsymbol{u}_L\left(\boldsymbol{x_0}, s\right)\, \mathrm{d}s. \tag{1.1}$$

However, the velocity in the Lagrangian description is equivalent to the velocity in the Eulerian description at a position $\boldsymbol{x}(\boldsymbol{x_0}, t)$. Taking the Taylor expansion, we see that

$$\boldsymbol{u}_L\left(\boldsymbol{x_0}, t\right) = \boldsymbol{u}\left(\boldsymbol{x}, t\right) = \boldsymbol{u}\left(\boldsymbol{x_0}, t\right) + \left(\int_0^t \boldsymbol{u}_L\left(\boldsymbol{x_0}, s\right)\, \mathrm{d}s\right) \cdot \nabla\boldsymbol{u} + \cdots \tag{1.2}$$

It is seen that there is a (first order) drift term, arising from the mismatch between the two velocities. If the motion is periodic, we can time-average over one period of oscillation (here denoted by angular brackets $\langle\cdots\rangle$) to find the Stokes drift velocity

$$\boldsymbol{u}_S = \left\langle \left(\int_0^t \boldsymbol{u}\, \mathrm{d}s\right) \cdot \nabla\boldsymbol{u} \right\rangle. \tag{1.3}$$

Understanding drifts arising from waves overlying a coral reef is important in modelling the hydrodynamics of coral reefs, with applications to the ecology of the reef (Monismith [3]) and, by extension, the recovery of damaged reefs which are often colonized by turf algae (Koehl *et al.* [4] and Roth *et al.* [5]). Indeed, there is some precedent for modelling flow above a coral reef, treating the porous layer as a lower boundary condition with drag (e.g. Rosman & Hench [6]). Additional investigations along this line, looking at the effects of friction between motion due to ocean waves and coral formations, is presented in Rogers *et al.* ([7–10]). The current paper, on the other hand, presents a new model for the associated Stokes drift due to a homogeneous, saturated, coral reef layer underlying an inviscid fluid, and derives expressions for the Stokes drift velocity both above and within the reef layer. Simplifications have been made—assuming the porosity is homogeneous in a coral reef of constant depth and that the top of the coral layer is flat, for example—to get a first-order picture of the Stokes drift in this situation, which we show has an important vertical component. While we are able to cite agreement with some field observations, there are a number of further details that will be included in the future to make the model more closely applicable to the many different coral reefs around the world.

We note that the porous layer underlying the system damps any waves on the fluid surface, and, in doing so, introduces a new vertical component of drift, which allows for an important net exchange of fluid between the two layers. This effect is most clearly seen when considering the paths of particles in the fluid, as in figure 1. Because the wave amplitudes decrease with horizontal distance travelled, the maximum vertical velocities attained over a period of wave motion also decrease, and therefore a fluid particle no longer has an oscillating vertical position. This effect was not noted by Stokes [1], because in the geometry he considered, the waves were undamped.

It is important to note that the model implemented here is invalid for fluid flow very close to the interface between the porous layer and the fluid which lies above it. This is because an individual fluid parcel may cross between flow regimes during a single wave oscillation. The only way in which we can see the behaviour of fluid near this interface is by producing plots of their paths, as in figure 1, and in the electronic supplementary material.

However, a model treating the coral reef layer as a homogeneous block underlying the ocean needs extension in order realistically to describe coral reefs, most noticeably because the reef permeability is not constant throughout the entire reef. In particular, an analysis with a constant permeability provides no way of considering stratified systems, where porous layers of different

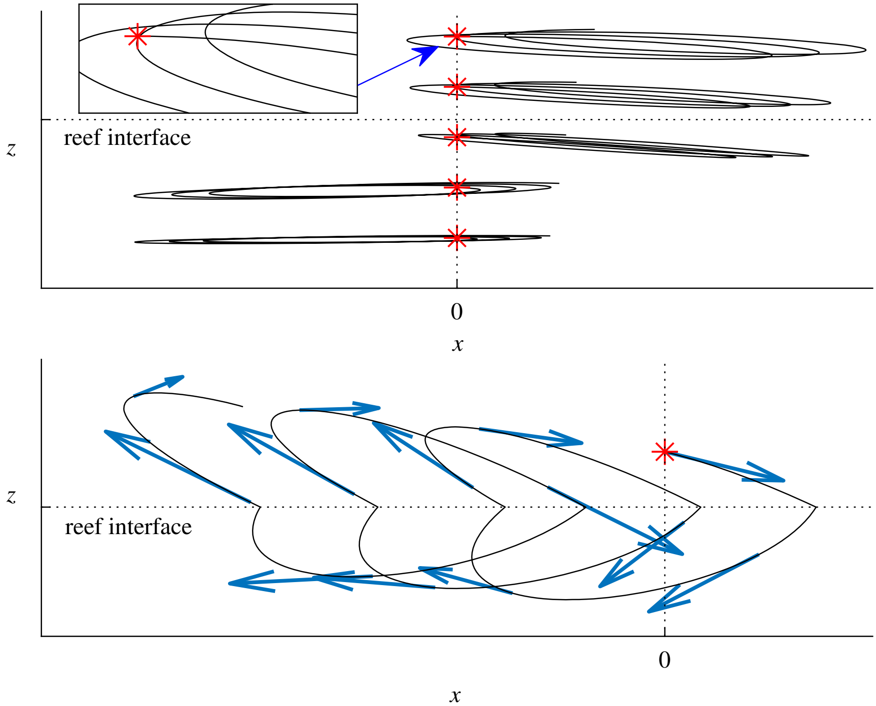

Figure 1. Plots of the paths of individual fluid 'particles' in the case of a homogeneous porous layer—the net vertical drift effect is clearly apparent in this case. This can be seen more clearly in the inset region, showing that the periodic paths are not closed orbits. The lower plot shows the path of a particle which crosses between layers, an indication of the dramatic drift effect due to the large differences between flow speeds in the two layers. Note also that, in this specific case, the discrepancy of flow speeds in the two layers produces a drift in the opposite direction to the wave propagation; this is only the case near the interface and is seen also in figures 6 and 7. (Online version in colour.)

permeabilities may overlay one another. An important example of this, discussed in Koehl *et al.* [4], involves a layer of algal turf overlying the coral reef, as is often the case after damage to a coral reef (Roth *et al.* [5]). We consider such three-layer systems, and note that the wavenumber of the surface waves is only affected by the permeability of the layer directly interfacing with the overlying fluid, that is to say, the upper porous layer. Therefore, it follows that the vertical drift effect mentioned above and discussed in depth by Webber *et al.* [11] depends solely on the permeability of the algal layer, meaning that such layers can have dramatic effects on the net exchange between water overlying the reef and the reef itself. An understanding of the differences that algal turf makes to the hydrodynamics of a coral reef is of increasing importance as such turf layers become more common, a phenomenon described in an ecological context by Tebbett & Bellwood [12].

2. Physical model

A two-layer model is introduced to simulate a coral reef in a shallow sea, comprising a porous layer of depth $D-d$ and vertically varying permeability $\kappa(z)$ underlying fluid of undisturbed uniform depth d. The surface of the fluid is described by $z=\eta(x,t)$, and there is an impenetrable barrier at $z=-D$. A diagram of the model is shown in figure 2.

The flow in the porous layer is governed by Darcy's Law, the use of which is well-established both in cases where the permeability is a constant and where it is spatially varying (Bear [13],

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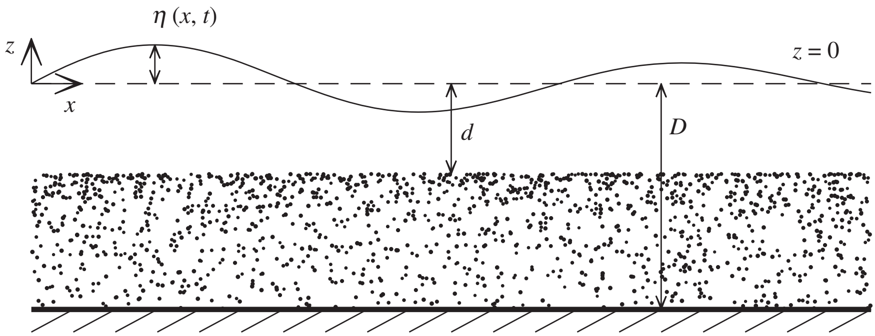

Figure 2. A diagram of the physical set-up, with the porous layer underlying water, on the surface of which are waves with mean vertical position at $z = 0$. Note the vertical variability of the permeability of this porous layer.

Hinton & Woods [14] and Ryoo & Kim [15]), leading to a velocity field of the form

$$\boldsymbol{u}\left(x,z,t\right) = -\frac{\kappa\left(z\right)}{\mu}\nabla p\left(x,z,t\right). \tag{2.1}$$

We could introduce a κ here that depends also on x, but this leads to issues matching at the boundary, as the resulting dispersion relation would need to be much more complicated. A further extension of this model would be to consider anisotropic permeability, where the reef permeability in the horizontal direction is different from that in the vertical direction. However, for the isotropic and horizontally invariant model considered, in the upper layer of the fluid, the disturbance on the surface is taken to have the form

$$\eta\left(x,t\right) = \mathrm{Re}\left\{A\mathrm{e}^{\mathrm{i}(kx-\omega t)}\right\}, \tag{2.2}$$

where A is the amplitude of the waves, and k the wavenumber, which is taken to be a complex number $k = k_R + \mathrm{i}k_I$ to allow for damping of the waves by the reef with horizontal distance travelled.

It is important to note that we are matching an inviscid layer which interacts with a viscously dominated layer below. Darcy's Law applies only in the lower regime, where the Reynolds number is small, while the flow above the porous layer is a high Reynolds number flow. Because of this difference, some care needs to be taken with the matching conditions. Working at the interface of these two layers, we follow the lead of Levy & Sanchez-Palencia [16] and Chen & Chen [17] to match both the normal components of velocity in the two layers (to impose mass conservation) and the pressure across the interface.

3. Deriving Stokes drift velocities

(a) Velocity fields

From these starting assumptions, the velocity field in the upper layer above the porous medium can be derived as having the form

$$\boldsymbol{u}\left(x,z,t\right) = \mathrm{Re}\left\{\alpha k e^{\mathrm{i}(kx-\omega t)}\left[\mathrm{i}\cosh\left(kz+\beta\right),\sinh\left(kz+\beta\right)\right]\right\}, \tag{3.1}$$

by remarking that the velocity potential ϕ, where $\boldsymbol{u} = \nabla\phi$, must be a harmonic function to satisfy incompressibility (e.g. Phillips [2]). Here α and β are undetermined, possibly complex, constants.

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Within the porous layer, a similar argument for the pressure field allows us to derive

$$\boldsymbol{u}\,(x,z,t) = -\frac{\kappa\,(z)}{\mu}\mathrm{Re}\left\{\gamma k \mathrm{e}^{\mathrm{i}(kx-\omega t)}\,[\mathrm{i}\cosh\,(kz+\delta),\sinh\,(kz+\delta)]\right\}. \tag{3.2}$$

The constants α, β, γ and δ are determined by the relations

$$\beta = \operatorname{arctanh}\left(\frac{\omega^2}{gk}\right), \tag{3.3a}$$

$$\alpha = -\mathrm{i}gA/\,(\omega\cosh\beta)\,, \tag{3.3b}$$

$$\gamma = \mathrm{i}\rho\omega\alpha\cosh\,(\beta-kd)/\cosh k\,(D-d) \tag{3.3c}$$

and

$$\delta = kD \tag{3.3d}$$

arising from matching and boundary conditions, and hence the dispersion relation is

$$\kappa|_{z=-d}\,\omega\tanh k\,(D-d) = \mathrm{i}\nu\tanh\left(\operatorname{arctanh}\left(\frac{\omega^2}{gk}\right)-kd\right). \tag{3.3e}$$

More details on the derivation of this expression are given in the forthcoming companion paper by Webber & Huppert [11]. Considering the case of no porous layer ($d\to D$), we recover the familiar dispersion relation $\omega^2 = gk\tanh kD$ first derived by Airy (see Craik [18]) and reproduced in equation (14) of Stokes [1].

Note here that the constants' values depend solely on the value of the permeability at the interface between the porous medium and the fluid overlying it, and not on any values of permeability within the porous layer. This can be interpreted as the fact that the flow above the porous layer is not at all affected by the permeability profile of the porous layer underlying it in this model. This is an effect which arises from the independence of any of the matching conditions on velocity derivatives, as we match only pressures and not stresses at the interface.

(b) Stokes drift velocities above the reef

With the above velocity fields in mind, we see that the velocity field above the reef is of an identical form to that for a fluid layer with no porous bed, as detailed by Stokes [1], albeit with different values of constants. Therefore, the Stokes drift velocities can be easily found. Using the result of equation (1.3), where the brackets $\langle\cdots\rangle$ denote time-averaging over a period $T=2\pi/\omega$ of wave motion,

$$\langle f\rangle = \frac{\omega}{2\pi}\int_0^T f\,\mathrm{d}s, \tag{3.4}$$

we determine that the upper-layer Stokes drift velocity, $\boldsymbol{u}_S^{(U)}$, is given by

$$\boldsymbol{u}_S^{(U)} = \frac{|k|^2\,|\alpha|^2\,\mathrm{e}^{-2k_I x}}{2\omega}\Big[k_R\cosh\,(2\mathrm{Re}\,\{kz+\beta\}), k_I\sinh\,(2\mathrm{Re}\,\{kz+\beta\})\Big]. \tag{3.5}$$

(c) Stokes drift velocities within the porous layer

Unlike the case above the porous layer, the velocity field within the reef (equation (3.2)) has explicit dependence on the depth-dependent permeability $\kappa(z)$. Defining the horizontal and vertical components of $\boldsymbol{u}$ to be u and v, respectively, we determine the components of $\nabla\boldsymbol{u}$

$$\nabla u = \frac{1}{\mu}\Big[\kappa\,(z)\,\mathrm{Re}\left\{k^2\gamma\cosh\,(kz+\delta)e^{\mathrm{i}(kx-\omega t)}\right\}, \\ -\kappa\,(z)\,\mathrm{Re}\left\{\mathrm{i}k^2\gamma\sinh\,(kz+\delta)e^{\mathrm{i}(kx-\omega t)}\right\} - \frac{\partial\kappa}{\partial z}\mathrm{Re}\left\{\mathrm{i}k\gamma\cosh\,(kz+\delta)e^{\mathrm{i}(kx-\omega t)}\right\}\Big] \tag{3.6a}$$

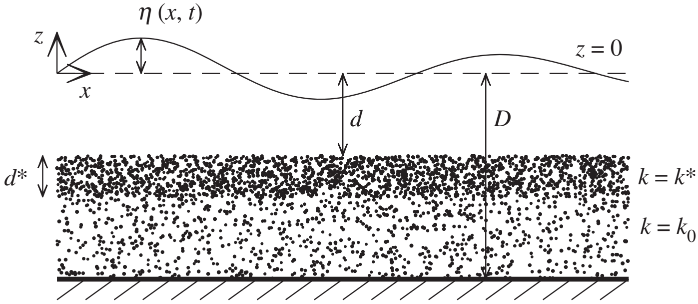

Figure 3. A diagram of the physical set-up for a two-layer porous medium, where the two layers have different permeabilities.

Table 1. Parameters used to produce the sample plots in the case of varying permeability. A full justification of these choices is contained in Webber & Huppert [11], but they are based on field measurements of reefs in Kaneohe Bay, Oahu, Hawai'i (private communication with Koehl, 2019).[a]

parameter	value
amplitude A	0.1 m
frequency ω	$2\,\mathrm{s}^{-1}$
horizontal offset x	0 m
total depth D	1.6 m
depth of overlying fluid d	0.6 m
dynamic viscosity μ	$1\times 10^{-3}\,\mathrm{kg\,m^{-1}\,s^{-1}}$
fluid density ρ	$1.04\,\mathrm{m}^{-3}$
gravitational constant g	$9.81\,\mathrm{m\,s^{-2}}$
permeability of coral reef κ_0	$5\times 10^{-7}\,\mathrm{m}^2$

[a] The value of κ_0 was determined using samples of *Porites compressa* coral from Kaneohe bay, and these samples were seen to have permeabilities that were approximately isotropic.

and

$$\nabla v = -\frac{1}{\mu}\Big[\kappa\,(z)\,\mathrm{Re}\left\{\mathrm{i}k^2\gamma\,\sinh\,(kz+\delta)\mathrm{e}^{\mathrm{i}(kx-\omega t)}\right\},$$

$$\kappa\,(z)\,\mathrm{Re}\left\{k^2\gamma\,\cosh\,(kz+\delta)\mathrm{e}^{\mathrm{i}(kx-\omega t)}\right\}+\frac{\partial\kappa}{\partial z}\mathrm{Re}\left\{k\gamma\,\sinh\,(kz+\delta)\mathrm{e}^{\mathrm{i}(kx-\omega t)}\right\}\Big]. \tag{3.6b}$$

Also,

$$\int_0^t \boldsymbol{u}\,\mathrm{d}s = \frac{\kappa\,(z)}{\omega\mu}\mathrm{Re}\left\{\left[e^{\mathrm{i}kx}k\gamma\,\cosh\,(kz+\delta)\left(e^{-\mathrm{i}\omega t}-1\right),\,-\mathrm{i}e^{\mathrm{i}kx}k\gamma\,\sinh\,(kz+\delta)\left(\mathrm{e}^{-\mathrm{i}\omega t}-1\right)\right]\right\}, \tag{3.7}$$

and thus we find the Stokes drift velocity, from equation (1.3), to be

$$u_S^{(P)} = \frac{|k|^2\,|\gamma|^2\,\kappa\,(z)}{2\omega\mu^2}e^{-2k_I x}\left(k_R\kappa\,(z)\,\cosh\,(2\mathrm{Re}\,\{kz+\delta\})+\frac{1}{2}\frac{\partial\kappa}{\partial z}\sinh\,(2\mathrm{Re}\,\{kz+\delta\})\right) \tag{3.8a}$$

and

$$v_S^{(P)} = \frac{|k|^2\,|\gamma|^2\,\kappa^2\,(z)}{2\omega\mu^2}e^{-2k_I x}k_I\,\sinh\,(2\mathrm{Re}\,\{kz+\delta\}). \tag{3.8b}$$

Remarkably, this result shows that the vertical Stokes drift velocity is not affected by vertical inhomogeneities in the porous layer, and the effect of the $\partial\kappa/\partial z$ term is only on the horizontal drift. This may initially appear surprising, but is a direct result of the requirement of continuity of

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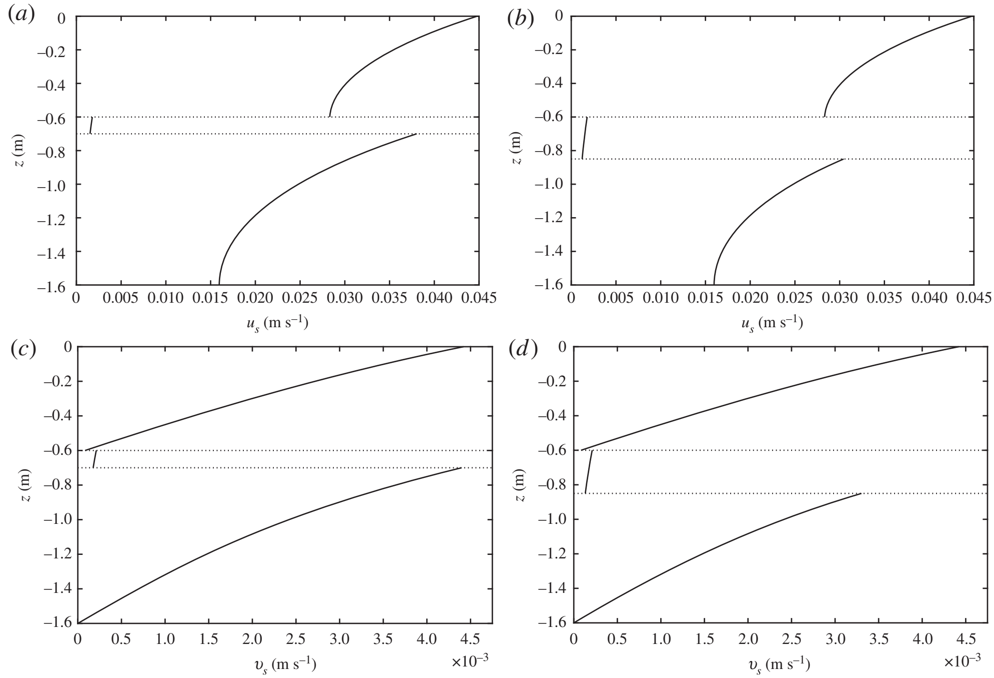

Figure 4. Plots of the horizontal and vertical Stokes drift velocities when the thickness of the upper porous layer is changed from $d^* = 0.1$ m (*a*,*c*) to $d^* = 0.25$ m (*b*,*d*)—note that the velocities in the lower layer remain largely similar in magnitude. Here, we have taken $\kappa^* = 1 \times 10^{-7}$ m^2.

vertical velocities at the interface, while an equivalent condition for horizontal velocities between the two layers is neither imposed nor necessary.

4. Examples with varying permeability

From the results in §3, some results common to all porous layers can be summarized. Firstly, the wavenumber is only dependent on the value of the permeability at the interface and not on its values within the porous medium—therefore, the amount of damping of the waves is also quantified solely by the permeability at the interface.

The apparent surprise of this situation results from the transition from high Reynolds number flow above the porous layer to viscously dominated, low Reynolds number flow within. This prediction might be different if we considered a model using the adaptation of Darcy's Law posed by Forchheimer [19], or that posed by Brinkman [20] (specifically designed for transitions from non-Darcy to Darcy flow). For example, if the permeability decreased with depth from being effectively infinite, the reef would have no damping effect on the waves—we would need to consider further drag effects in this case because Darcy's Law would no longer properly apply in the porous layer. However, provided the magnitudes of velocities are sufficiently small, we legitimately continue to use the Darcy model.

Additionally, there is no effect on the vertical Stokes drift velocity from the varying permeability—again, it is only affected by the value of the permeability at the interface. This is a consequence of the continuity of vertical velocities at the interface.

(a) Three stratified layers

We now consider a model where there is a layer of depth d^* and permeability κ^* occupying the region $-(d + d^*) < z < -d$ and a layer of permeability κ_0 occupying the region $-D < z < -(d + d^*)$,

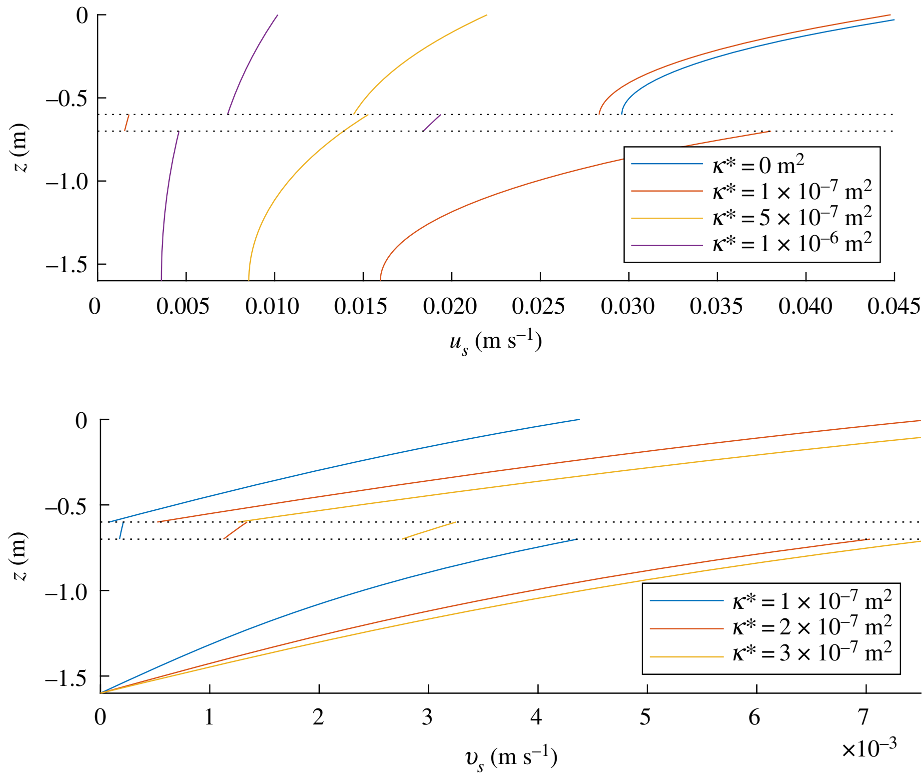

Figure 5. The horizontal and vertical Stokes drift velocities for differing permeabilities of the upper porous layer. Note that there is no drift in the lower layer for an impermeable boundary ($\kappa^* = 0$). Recall that the permeability of the lower porous layer is $\kappa_0 = 5 \times 10^{-7}$ m^2. (Online version in colour.)

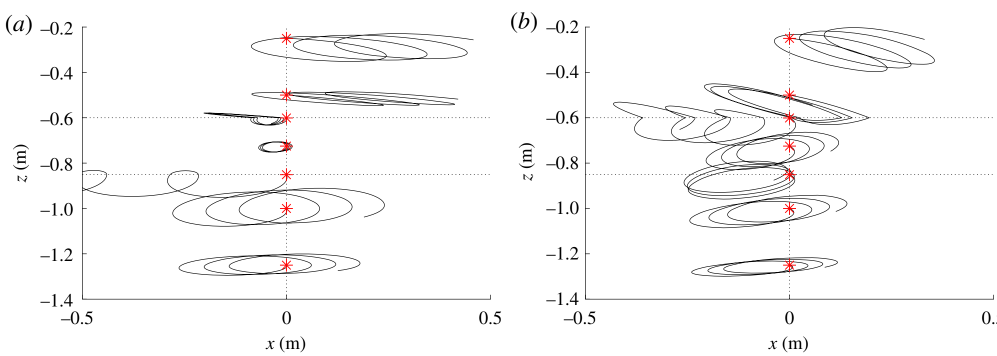

Figure 6. Plots of individual fluid particle paths in the case where $\kappa^* = 1 \times 10^{-7}$ m^2 (*a*) and $\kappa^* = 4 \times 10^{-7}$ m^2 (*b*). Note the rapid drift near the interfaces due to flow being very different in the layers which meet. (Online version in colour.)

as illustrated in figure 3. For the purposes of explicit calculation, we will consider the physical parameters shown in table 1.

(i) Varying thickness of upper porous layer

If we fix the permeability of the upper porous layer and vary its thickness d^*, we see that the dispersion relation of equation (3.3*e*) is unchanged—it only depends on the permeability at the

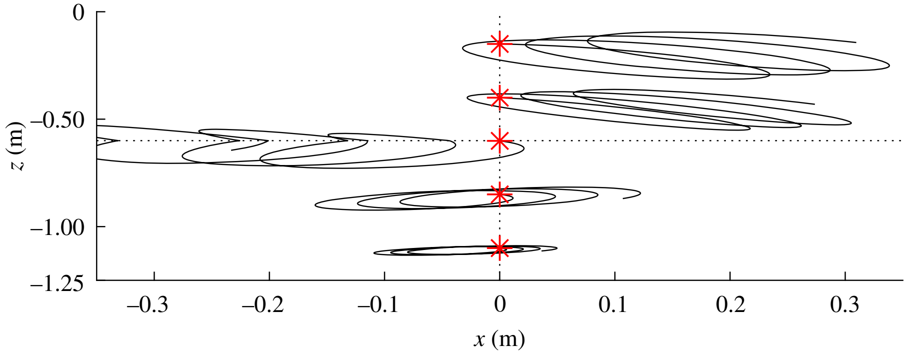

Figure 7. Plots of individual particle paths in the case where the permeability of the porous layer varies as described by equation (4.1). (Online version in colour.)

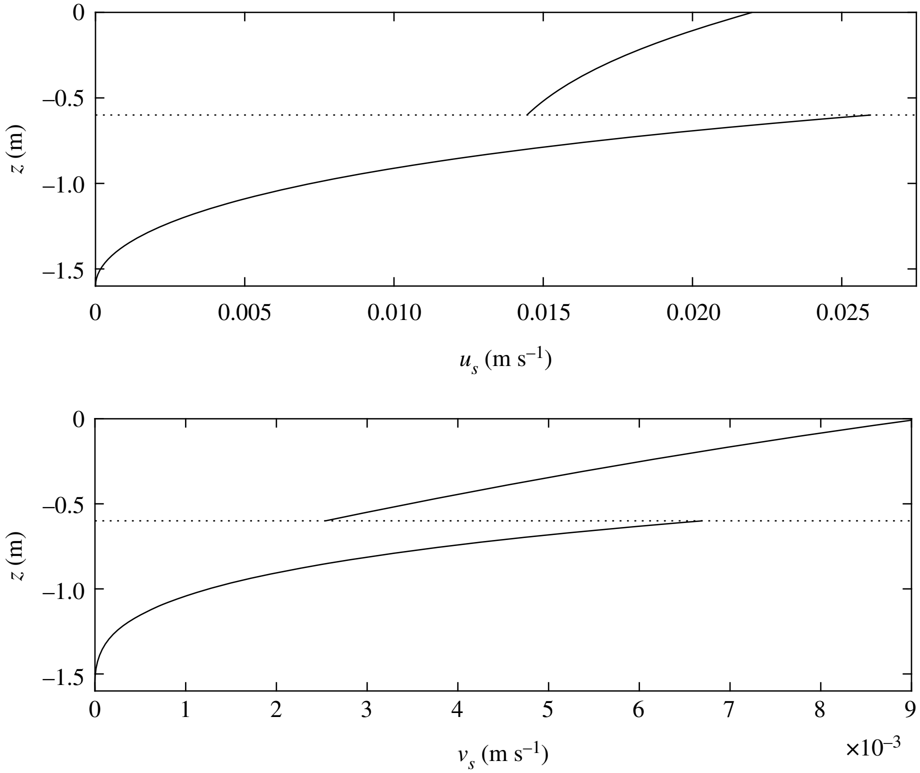

Figure 8. The horizontal and vertical Stokes drift velocities for a porous layer with linearly-varying permeability, as in equation (4.1).

interface which is a constant κ^*. Likewise, all of the other constants of equation (3.3) remain the same. Furthermore, inspecting the form of the Stokes drift velocity expressions in both layers, the drift velocities at a fixed value of z in the lower porous layer are unchanged by modifying the thickness of the upper porous layer, a fact shown in figure 4.

(ii) Varying permeability of the upper porous layer

If we instead fix $d^* = 0.10\,\text{m}$, and vary the permeability of the upper porous layer, we obtain effects throughout the profile, because the dispersion relation will change. Letting $\kappa^* \to 0$, we recover the classical results for a fluid of depth d, as derived by Stokes [1], because the upper layer becomes an impermeable boundary. The vertical drift is therefore zero in this case because the waves are not damped. Vertical drift effects increase with the increasing permeability of this layer, because more and more flow can be exchanged between the two flow regimes, as shown in figure 5.

The behaviour near the interfaces between porous layers is investigated in figure 6—plotting particle paths shows that particles that cross between flow regimes, precisely at the interface between layers, drift rapidly, because they spend some of their oscillatory period in a region with a much smaller magnitude of velocity—as is manifested in the paths plotted in figure 6. Such drifts are not captured in the expressions for the Stokes drift that we have derived above because they are based on the assumption that fluid stays in one regime for the entire oscillation.

(b) Continuously varying permeability

We now consider the case where the permeability of the porous layer is not straightforwardly separated into two layers, but instead varies continuously. The simplest case that we consider here is a porous layer impermeable at its base with permeability increasing to a given value at the interface with the overlying fluid. The permeability is thus assumed to be

$$\kappa(z) = \frac{\kappa_0}{D-d}(z+D) \quad \text{for } z \leq -d. \tag{4.1}$$

Now taking $\kappa_0 = 5 \times 10^{-7}\,\text{m}^2$, $D = 1.6\,\text{m}$ and $d = 0.6\,\text{m}$, we plot particle paths, as in figures 1 and 6—the results of which are shown in figure 7. Aside from the permeability, all of the physical parameters are the same as in table 1.

As would be expected, the Stokes drift velocity—both horizontally and vertically—is seen to tend to zero as we approach $z = -D$. This is shown more clearly in figure 8.

5. Conclusion

We have shown that the methods originally derived by Stokes [1] in his seminal paper of 1847 can be extended to more complex cases, where there is both damping of waves and vertical inhomogeneity. The same original approach is still valid, but some novel effects, including a vertical drift due to damping (the behaviour of which is to be elucidated in more detail by Webber & Huppert [11]), are seen to arise.

It is seen that allowing for variation of the permeability of a porous layer with depth does not change the nature of the velocity field of the model, because the dispersion relation for the waves depends only on the value of the permeability at the top of the porous layer (equation (3.3*e*)). The only change is a new dependence on z, which is taken into account when calculating Stokes drift velocities.

The form of these velocities is seen to be similar to those considered by Stokes [1], save for an additional term in the horizontal drift within the porous layer, which is dependent on $\partial\kappa/\partial z$, the rate of change of permeability with depth. This model does, however, lead to the surprising result that even a very thin layer of high permeability overlying a block of lower permeability has the same damping effect on the waves as a homogeneous block of the higher permeability would. It is suggested that further investigations might consider Brinkman (and/or Forchheimer) drag effects in the porous layer as an additional potential mechanism for damping.

The first situation we considered is a porous layer of uniform permeability that overlies a second porous layer of uniform permeability—considered here as an analogue for an inflexible calcareous algal layer that overlies a coral reef. It is noted that varying the thickness of this upper layer has no effect on Stokes drift velocities above or below (i.e. in the upper fluid layer or the porous bed), and only affects the drift velocity in the layer itself.

The second considered situation is of a linearly decreasing permeability with depth, considered here as an analogue of a coral reef with coral density increasing with depth. This has the expected result of Stokes drift velocities decreasing (and indeed tending to zero if the porous layer base is impermeable) as we approach the base of the model.

However, as already mentioned, we acknowledge the possible shortcomings of this model where changes in permeability are rapid, or drag effects are otherwise significant. We suggest that this could be overcome through use of a Brinkman model for the porous layer, where we instead take

$$u(x,z,t) - \underbrace{\beta\nabla^2 u(x,z,t)}_{\text{Brinkman term}} = -\frac{\kappa(z)}{\mu}\nabla p(x,z,t), \tag{5.1}$$

with β an effective viscosity parameter. The authors conjecture that the low characteristic velocities of fluid flow in and through coral reefs, however, suggest that any drag effects would be sufficiently small to be ignored to a good approximation.

Coral reefs are complex and highly inhomogeneous structures. Measurements made of *Porites compressa* corals show that, although permeability appears relatively isotropic on small scales, it is clear that there are limitations in modelling such complex structures as simple porous layers. Initial results, however, to be published in Webber & Huppert [11], suggest that this model's predictions of drift velocities of the order of $1\,\mathrm{cm\,s^{-1}}$ agree with field measurements.

There are numerous ways in which this model could be extended to model different reef situations that appear in nature. Perhaps most apparently, we could consider interfaces which are not parallel to the impenetrable floor—in the simplest case, this could take the form of sloping interfaces which remain a straight line, and the model might be further reworked in the case where there are concavities and convexities in the bounding surface. In the absence of detailed data on the reef layout, however, comparison with field data would be somewhat more difficult.

It remains to compare this model with field measurements—however, with no data available on the structure of the reef or overlying algal layers, this is not yet possible. Furthermore, this model neglects any compression or other deformation of the algal turf layer, something only valid for some stiffer algae (Connell *et al.* [21]). In spite of this, it is seen that this simplified model provides a starting point for further investigation of these drift velocities and the novel vertical drifts that appear from the inclusion of a porous layer.

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Authors' contributions. All authors contributed to the writing and revision of the manuscript.
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