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A note on shock diffraction by a supersonic wedge

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An earlier treatment of the diffraction of a shock wave advancing into a region of uniform flow, based on Chisnell's (1965) extension of Whitham's (1957) rule for shock diffraction, is corrected for an algebraic error and then compared with an analogous treatment based on the more recent extension derived by Whitham (1968). The basis for comparison is the pressure just behind a shock wave that is diffracted by a thin wedge travelling at supersonic speed. The approximation provided by Whitham's extension is both simpler than, and typically superior to, that provided by Chisnell's extension (although the numerical differences are small in the Mach-number régime considered).

1. Introduction

We consider the diffraction of an approximately uniform, plane shock wave that moves into a region of uniform, plane flow, modifying an earlier analysis by Miles (1965). This analysis was based on Chisnell's (1965) extension of Whitham's (1957) treatment of a shock wave moving into a uniform, quiescent region and was applied to the calculation of the pressure just behind a diffracted shock wave on a wedge which penetrates that shock wave at supersonic speed. The result was compared with that inferred from Smyrl's (1963) solution of the linearized boundary-value problem. Since then, Whitham (1968) has proposed that his original treatment be extended through a Galilean transformation, with results that differ from those based on Chisnell's modification. Moreover, we have found that Miles's (1965) calculation of the initial angle of diffraction of the shock contained an error.

We present here a comparison of the alternative approximations to the wedge pressure just behind the diffracted shock wave, based on the respective methods of Chisnell (1965) and Whitham (1968).

2. Shock-diffraction approximations

Assuming that all velocities are approximately parallel (so that we may neglect the squares of the angles of inclination), we consider an approximately uniform, plane shock wave that moves to the right with relative Mach number m into a uniform, plane flow of Mach number M. We require the change in m, say δm, associated with a small change, say $\delta\theta$, in the angle of inclination of the shock

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($\delta\theta > 0$ implies that the shock is locally concave with respect to the uniform flow on the right; see figure 1 of Miles 1965).

Employing Chisnell's (1965) extension of Whitham's method to determine the variation of m with angle of diffraction of the shock, we obtain (Miles 1965)

$$\left(\frac{\delta m}{\delta\theta}\right)_M = \left\{\frac{m+M}{m+L(m)M}\right\}^{\frac{1}{2}}\left(\frac{\delta m}{\delta\theta}\right)_0 \equiv \left(\frac{\delta m}{\delta\theta}\right)_1, \tag{1}$$

where: $(\delta m/\delta\theta)_0$ is given by Whitham's [1957, equation (22)] 'shock-shock' relation for a shock moving into a quiescent region ($M = 0$); the function $L(m)$ is given by Miles (1965) and decreases from 1 at $m = 1$ through a very flat minimum of approximately 0·7774 near $m = 3$ to 0·7848 at $m = \infty$ (for $\gamma = 1{\cdot}4$). Employing Whitham's method in a reference frame for which the fluid velocity ahead of the shock is zero (Whitham 1968), we obtain, with no further assumptions,

$$\left(\frac{\delta m}{\delta\theta}\right)_M = \left(\frac{\delta m}{\delta\theta}\right)_0 \quad \text{for all } M. \tag{2}$$

We remark that (1) reduces to (2) for $m \to 1$, $m \to \infty$, and $M \to 0$, and that the maximum discrepancy between the two results is roughly 30 %.

3. Numerical comparisons

Recalculating the ratio of the initial angle of diffraction of the shock wave to the wedge angle, we obtain (where ϵ is the shock-diffraction angle and α is the wedge deflexion angle; see Miles 1965, figure 2)

$$\begin{aligned}\epsilon/\alpha = M\{&m(M+m)+\sigma[(1-\kappa)^{-1}(M+m)^2(m^2+1)-m^2(m^2-1)(M^2-1)]\}^{-1}\\ &\times\{M+m-\tfrac{1}{4}(\gamma+1)M(1-\kappa m^2)+2m\sigma\\ &\times[\tfrac{1}{4}(\gamma+1)m(2m+M-M^2m)-\tfrac{1}{2}(\gamma-1)(M+m)^2(1-\kappa)^{-1}]\},\end{aligned} \tag{3}$$

where

$$\sigma = \tfrac{1}{2}m^{-2}(1+\gamma\kappa m^2)^{\frac{1}{2}}(1-\kappa)^{\frac{1}{2}}\{(m+M)^2-(1-\kappa)(m^2-1)(M^2-1)\}^{-\frac{1}{2}}. \tag{4}$$

This differs from the erroneous result reported by Miles [1965, equation (3.2)], but can be shown to be in agreement with the corresponding result implied by Smyrl's (1963) formulation. Typical numerical results are plotted in figures 1 and 2.

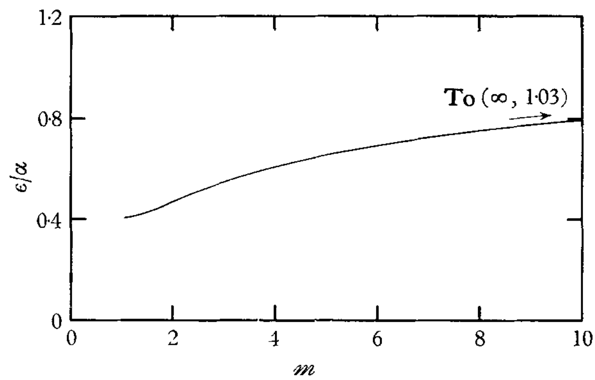

FIGURE 1. The ratio of the shock diffraction angle ϵ to the wedge deflexion angle α for $M = 2$ and $\gamma = 1{\cdot}4$.

Repeating the approximate calculation of the pressure just behind the diffracted shock wave on the wedge (Miles 1965, §4), we find that the result based on $(\delta m/\delta\theta)_0$ is significantly closer to the exact result than that based on $(\delta m/\delta\theta)_1$

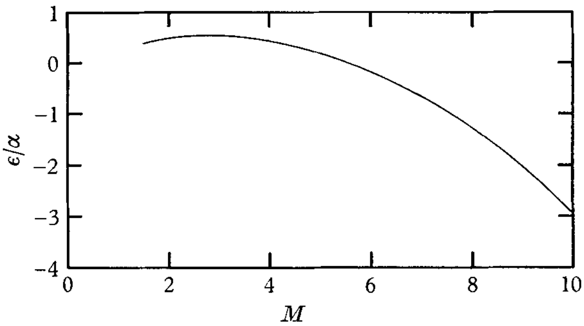

FIGURE 2. The ratio of the shock diffraction angle ϵ to the wedge deflexion angle α for $m = 2$ and $\gamma = 1{\cdot}4$.

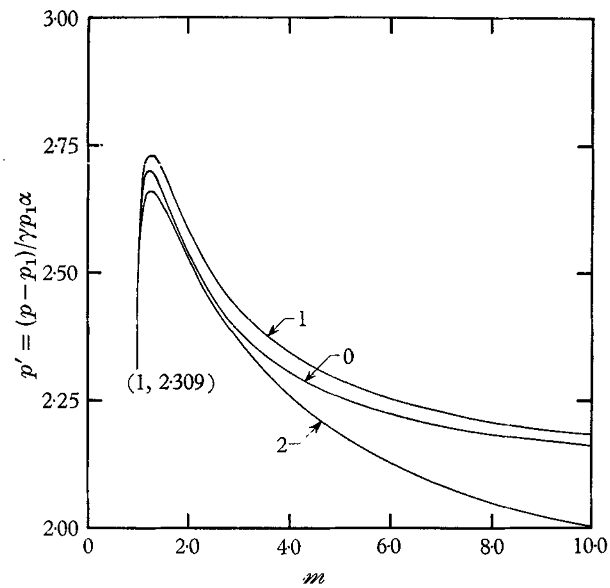

FIGURE 3. The relative pressure on the wedge, just behind the diffracted shock wave, for $M = 2$ and $\gamma = 1{\cdot}4$. Curves 0 and 1 are based on $(\delta m/\delta\theta)_0$ and $(\delta m/\delta\theta)_1$, respectively; curve 2 is based on the linearized boundary-value problem. p_1 is the undisturbed pressure behind the incident shock wave.

for most m and M.† Typical results are plotted in figures 3 and 4, from which it can be seen that the use of either value for $(\delta m/\delta\theta)$ leads to an error of approximately 5 % (note that in both these figures the zero of the vertical axis has been suppressed and the vertical scale exaggerated in order to emphasize the difference between the different formulations).

† There do exist values of m, M for which the result based on $(\delta m/\delta\theta)_1$ is closer to the exact result than that based on $(\delta m/\delta\theta)_0$, but for these cases either formulation typically leads to an error of less than 1 %.

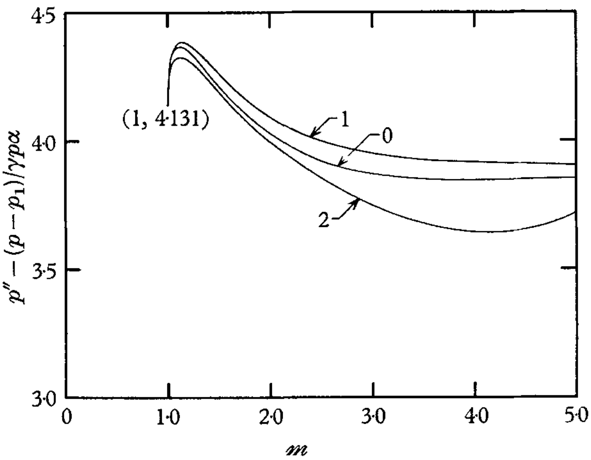

FIGURE 4. The pressure on the wedge, just behind the diffracted shock wave, for $M = 4$ and $\gamma = 1{\cdot}4$. Curves 0 and 1 are based on $(\delta m/\delta\theta)_0$ and $\delta(m/\delta\theta)_1$, respectively; curve 2 is based on the linearized boundary-value problem.

4. Conclusion

We infer from this comparison that Whitham's original (1957) formulation typically provides a more accurate approximation for the diffraction of a shock wave moving into an area of uniform flow than does Chisnell's (1965) generalization thereof.

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