Geological Fluid Mechanics

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1. The Earth

Since antiquity there has been consideration and speculation about the Earth on which we live. How did it form; how does it evolve; how can its riches be exploited? It was relatively easy to understand some of the fundamentals of the enveloping atmosphere, of order 10 km thick, because of its optical transparency. Some of these fundamentals and the ensuing consequences are described in Chapters X and Z by Linden and McIntyre. Satisfactory investigations of the oceans, which cover 70% of the globe to a mean depth of 4 km, were more difficult. Some of these are described in Chapter Y by Garrett. The "solid" Earth, of mean radius 6371 km, whose volume and mass greatly exceed that of either the atmosphere or the oceans (see Table 1), has been the most difficult to examine. In large part this is because almost all of the globe is inaccessible to direct observation. Inferences have to be drawn from observations at (or near) the Earth's surface, appropriately combined with theoretical reasoning. Fluid mechanics plays a considerable and ever increasing role in this investigation.

The relatively new subject of geological fluid mechanics is concerned with applying fundamental fluid-mechanical concepts to following the motion of the fluid material that upon either solidification or sedimentation become the rocks that make up the Earth. A full understanding of the subject comes from a combination of theoretical analysis, data from laboratory experiments and field observations; and this breadth of essential input is partly what has made development of the area so exciting in the last decades of the millennium.

Much of the subject is motivated by the motions of: magma – the geologist's term for fluid rock when within the Earth; or lava – the name for magma erupted from volcanoes at the surface of the Earth; or particulate suspensions in either the oceans or atmosphere due to a variety of natural processes including volcanic eruptions. Both chemically and physically, magma (and lava) can vary enormously from one location within (or on) the Earth to another. The major constituent, silica, can be as low as 45% or as high as 75%, which might be compared with the very much smaller range of the major constituents of the atmosphere, nitrogen and oxygen, or those of the oceans, water and salt. The viscosity varies over many orders of magnitude with both composition and temperature. A silica-poor, high-temperature magma may have a viscosity as low as 5×10^{-6} m^2 s^{-1}, while a silica rich, low temperature magma may have a viscosity of 10^8 m^2 s^{-1} or even more as it finally solidifies. In detail, many magmas are no doubt non-Newtonian, and the values above are but constructs. However, much can be learnt from purely Newtonian models of magma and lava flows and much, but definitely not all, of the material presented in this chapter is developed on the basis of Newtonian fluid mechanics, which is a good approximation in many cases.

An extensive terminology for magmas and lavas has developed, based mainly on their chemical composition. Some understanding of this (not always uniquely defined) terminology is necessary to read the geological literature. For the purposes of this chapter, however, it suffices to define two of the most common types. *Basalts* are magmas derived from the Earth's mantle and are composed of approximately 50% silica, have temperatures between 1100 and 1250°C, densities between 2.60 and 2.75 g cm^{-3} and viscosities between 0.003 and 0.1 m^2 s^{-1}. *Rhyolites* are magmas that can become granite on solidification within the Earth and originate from melting of the continental crust or by chemical evolution of basalt. Rhyolites have a silica content of between 70 and 75%, have temperatures between 700 and 1000° C, densities of approximately 2.3 g cm^{-3} and viscosities between 1 and 10^8 m^2 s^{-1}.

The formation of the Earth (and the other terrestrial planets) was a by-product of the formation of the Sun. The Sun, along with a number of stars, was formed by gravitational instability of the dense, rotating interstellar molecular medium. It is commonly believed that the terrestrial planets then grew by agglomeration of so-called planetesimals, which formed in the solar nebula and ranged in size from a few metres to a few thousands of kilometers. Calculations indicate that by this process a body of 10^{23} kg can result in about 10^5 years. Of the order of 100 such planetary embryos then merged in the next 10^8 years to form the early Earth. During this period giant impacts occurred between bodies of similar mass, which caused melting and reconfiguration of the Earth to great depths. One of these giant impacts is believed to have resulted in the orbital injection of material from which the Moon formed. These processes ceased about 4.5 x 10^9 years ago, the date recognized as the 'birth' of the Earth as we know it today.

It is generally believed that after each giant impact the Earth was left molten. Cooling, by radiation into space, first formed an outer crust, though no geological record remains of this early crust. The current continental crust formed later, beginning around 4 x 10^9 years ago, and is now typically between 25 to 40 km thick. Oceanic crust, which is much younger and has been frequently recycled, is only about 5 km thick. At the base of the crust lies the Mohorovičić discontinuity (known after its discoverer) or Moho for short. Its existence was identified by its ability to reflect and scatter seismic waves, an important tool in the investigation of the physical and chemical properties inside the Earth. The region between the Moho and a depth of 670 km is known as the upper mantle. At this depth a phase transition occurs and below this, to a depth of 2920 km, extends the lower mantle. The mantle is rich in silicates with the most common upper mantle mineral being olivine, which at its simplest is a solid solution of magnesium and iron silicate. Below the mantle is the iron-rich liquid outer core, as first announced by Jeffreys in 1926 (see figure 1). At the centre of the Earth there is a solid inner core which is closer to pure iron than even the outer core and currently extends to a radius of 1221 km. The existence of this solid inner core was first suggested in 1936, from seismic evidence, by Inge Lehmann, who died recently in her 105th year. This differentiation into a core dominated by iron and a mantle dominated by silica occurred when the Earth was molten and the heavy iron 'sank' to the centre.

Of course virtually no part of the Earth is static – from the inner core to the outer stratosphere it is almost all in motion, in some parts quite vigorous. The temperature of the actual centre of the Earth is currently somewhere between about 5000 and 6500° C (which represents a fascinatingly large range of uncertainty about a particular and important geophysical quantity). The temperature at the boundary between the inner and outer cores is thought to be about 250° C lower. It has been suggested that this temperature difference drives convection in the inner core, though there is as yet no observational evidence for this. As a consequence of the slow cooling of the Earth, the iron-rich, liquid outer core cools and part of it solidifies to form almost pure iron at the inner/outer core boundary to release fluid depleted in iron and hence less dense than its surroundings†. This is an archetypical situation for compositional convection (Huppert &

† Solidification takes place at the hotter surface of the boundary of the outer core, rather than at the approximately 1500° C cooler boundary with the mantle because of effects due to pressure. The amount of iron produced at the inner/outer boundary exceeds by about four orders of magnitude the total iron and steel production of mankind on the surface of the Earth; nevertheless the radius of the inner core increases at a rate estimated to be only about 1mm/year.

Sparks 1984) as described in Chapter W by Worster. The rising plumes, which may break up into blobs, drive strong convective motions in the rotating outer core and maintain the Earth's magnetic field, as described in Chapter P by Moffatt (and further in §6). Driven partly by heat losses from the core and partly by heat generation from radioactive decay in the interior of the mantle, there is a complicated, unsteady bulk motion in both the lower and upper mantle with typical velocities of a few centimetres a year – about as fast as one's fingernails grow.

These motions lead to an important concept in understanding the Earth, the theory of plate tectonics, which was developed in its full form in the mid 1960s. Nineteenth century scientists, notably Alfred Wegener, had developed the rudimentary notions of continental drift, but the suggestion that the solid rocks of the Earth behave like a fluid seemed so implausible that the notion of continental drift was not initially accepted. However, the idea that the hot crystalline rocks of the mantle could flow in convective currents slowly began to be appreciated, partly due to the influence of the great British geologist Arthur Holmes. The major breakthrough came in the mid 1960s with proof, from the magnetic signatures of volcanic rocks on the sea floor, that continents could spread apart at the mid-ocean ridges by sea-floor spreading. The concept of plate tectonics rapidly evolved as it was recognised that only the outermost 100 km or so of the Earth (the lithosphere) was cold and rigid, whereas the bulk of the Earth's mantle was sufficiently hot that, despite its crystalline nature, it could convect and flow. The Earth's surface is broken into lithospheric plates which are constantly moving apart and (elsewhere) colliding together as a consequence of motion driven both by their own high density compared to the deeper, hotter Earth and by convection in the Earth's interior. Volcanoes and earthquakes are found to be mostly located in great zones marking the plate boundaries. This Chapter is not the place to detail the excitement in the mid 1960s as the theory evolved, nor to describe exactly how plate tectonics works. An excellent account, however, can be found in Gubbins (1990). The additional point to note here is that there also exist in the mantle and the crust much more rapid, smaller scale fluid motions of enormous importance. The investigation of these motions are central to this Chapter.

Much of the above introduction is treated at length in standard books on Earth Sciences. Interested readers might like to dip into Anderson (1989), Brown, Hawkesworth & Wilson (1992) or Press & Siever (1986). A short description of how some of the current physical facts about the Earth were determined, with an unashamed bias towards contributions made by Cambrige geophysicists, is contained in Huppert (1998a).

One important way by which an Earth scientist learns about the physics and chemistry of the interior of the Earth is from volcanic eruptions. The episodic eruption of magma periodically at the Earth's surface immediately indicates that some parts of the 'solid' interior must be molten. Investigations have shown that there are large storage reservoirs of (at least partially) liquid rock known as magma chambers at depths of between a few to perhaps tens of kilometres beneath all volcanoes, both those that erupt on the continents and the many more numerous volcanoes on the floor of the oceans. The chambers may range in horizontal dimensions from hundreds of metres to more than a hundred kilometres and in vertical dimensions from tens of metres to many kilometres. Each magma chamber, which may contain as much as 10^5 km^3 of molten rock, possibly with a considerable crystal content, all at temperatures from 800°C to over 1200° C, acts as the energy source for the overlying volcano. Many interesting and important physical and chemical processes occur in a chamber, some of them leading to an eruption. The purpose of this chapter is to present a description of some of the fundamental processes that occur in these situations. An earlier review, from a different perspective, of the role played by fluid mechanics in geology is contained in Huppert (1986).

2. Fluid processes in magma chambers

The central problem in magma chamber dynamics is the determination of the physical (and chemical) evolution of a large body of cooling and crystallizing, multi-component liquid. This section aims to build a physical understanding of the fundamentals of magma chamber dynamics by describing sequentially a range of fluid processes.

2.1. *Conduction*

The cooling of magma against surrounding country rock is one important aspect of many processes. As an introductory, very much simplified, problem consider the half-space $z > 0$ at uniform temperature T_+ suddenly brought at time $t = 0$ into contact with the half-space $z < 0$ at a lower uniform temperature T_-. Each half-space could be either solid or liquid – but the effects of any motion are neglected if it is liquid. The temperature $T(z, t)$ must then adjust purely by the conductive transfer of heat. Assume, for simplicity, that the thermal diffusivity of the material of each half-space is identical and denoted by κ. The situation is then described by the one-dimensional thermal conduction equation

$$T_t = \kappa T_{zz} \tag{2.1}$$

with initial and boundary conditions

$$T = T_- \quad (z < 0) \quad \text{and} \quad T = T_+ \quad (z > 0) \qquad (t = 0) \qquad (2.2\text{a, b})$$

$$T \to T_- \quad (z \to -\infty) \quad \text{and} \quad T \to T_+ \quad (z \to \infty) \qquad (\forall t > 0) \qquad (2.3\text{a, b})$$

$$T \text{ and } T_z \text{ continuous at } z = 0 \qquad (t > 0)\ . \qquad (2.4\text{a, b})$$

Because there is no specified length-scale in the problem, the solution of the partial differential system (2.1)–(2.4) cannot be a function of the two independent variables z and t separately but there must be a *similarity* solution in terms of one variable which is a suitable combination of z and t. In this case a suitable similarity variable is $\eta = \frac{1}{2}z(\kappa t)^{-\frac{1}{2}}$, which transforms (2.1)–(2.4) into the ordinary differential system

$$\frac{d^2T}{d\eta^2} - \eta\frac{dT}{d\eta} = 0 \tag{2.5}$$

$$T \to T_- \qquad (\eta \to -\infty) \qquad \text{and} \qquad T \to T_+ \qquad (\eta \to \infty) \tag{2.6a, b}$$

$$T \text{ and } T_\eta \text{ continuous at } \eta = 0 \ . \tag{2.7a, b}$$

The solution of (2.5) to (2.7) (in terms of η) is

$$T(\eta) = \tfrac{1}{2}(T_+ + T_-) - \tfrac{1}{2}(T_+ - T_-)\text{erf }(\eta) \ , \tag{2.8}$$

where erf $(x) = 2\pi^{-\frac{1}{2}} \int_0^x e^{-u^2} du$ is the error function. The relationship (2.8) indicates that the temperature of the boundary, $T(0,t)$, immediately adjusts to the mean temperature of the two half-spaces and remains at that value. This statement is one of the best known quantitative relationships amongst geologists.

Consider now the slightly more complicated problem where the half-spaces $z > a$ and $z < -a$ are initially at T_-, while the material in $-a < z < a$ is initially at T_+ $(> T_-)$. Because of the introduction of the length scale a, the solution is not expressible in terms of one similarity variable, but nevertheless can easily be determined to be (Carslaw & Jaeger 1980, p. 54)

$$T(z,t) = T_- + \frac{1}{2}(T_+ - T_-)\left\{\text{erf }\frac{a-z}{2(\kappa t)^{\frac{1}{2}}} + \text{erf}\frac{a+z}{2(\kappa t)^{\frac{1}{2}}}\right\} \ , \tag{2.9}$$

which is graphed as a function of z for various values of $\kappa t/a^2$ in figure 2. The time scale for the temperature evolution by conduction is given by $t_c = a^2/\kappa$, values for which are given in Table 2 for various values of a with $\kappa = 0.01$ cm^2 s^{-1}, a representative value for magmas and surrounding country rocks. The variation is seen to be considerable. This simple model is of some applicability to the cooling of either lavas or of relatively long, thin intrusive sheets of magma in what are called dykes or sills by geologists†. But consideration of further physical processes is needed before it is applicable to (thicker) magma chambers. Two of the main ones are the effects of convection and crystallization in the magma, both of which are dominant processes in real magma chambers.

2.2. *Thermal Convection*

In order to analyze some of the important effects of purely thermal convection in magma chambers, Jaupart & Brandeis (1986) carried out a series of experiments in which a layer of initially hot silicone oil was confined between two rigid horizontal boundaries whose temperature at $t = 0$ was suddenly decreased and maintained constant, at typically some 20° less than the initial temperature of the isothermal oil. Based on an initial temperature difference $\Delta T_i \simeq 20°$C, the initial Rayleigh number $Ra_i = \alpha g \Delta T_i d^3/\kappa\nu$ was of the order of 10^8, where g is the acceleration due to gravity, α and ν are the coefficients of thermal expansion and kinematic viscosity respectively and d =10 cm was the distance between the boundary plates. At such high Rayleigh numbers the resulting convective motions are turbulent (c.f. Linden, Chapter L) and the heat transfer is dominated by the convective component. Convective plumes penetrated the fluid from a layer near the upper surface, leading to an almost isothermal interior while a thin, stagnant, thermally stable boundary layer evolved at the base.

With the employment of the famous four-thirds flux law for turbulent convection (Linden p. ??) and the equation of this heat loss to the rate of decrease of heat in the interior, the thermal balance, in terms of the uniform temperature $T(t)$ of the interior, becomes

$$cd\frac{dT}{dt} = -0.12k\left(\frac{\alpha g}{\kappa\nu}\right)^{\frac{1}{3}}(T - T_B)^{\frac{4}{3}} \ , \tag{2.10}$$

† Solutions to a whole series of conduction problems are presented in Carslaw & Jaeger (1980) and reviewed in the context of the Earth sciences by Jaeger (1968).

where T_B is the (maintained constant) temperature of the boundaries and c is the specific heat per unit mass. Nondimensionalising temperature with respect to the initial temperature difference and time with respect to the conduction time based on the thickness of the layer to introduce the variables

$$\theta(\tau) = [T(t) - T_B]/\Delta T_i \quad \text{and} \quad \tau = \kappa t/d^2 \equiv t/t_c \ , \tag{2.11a, b, c}$$

we can write the solution of (2.10) as

$$\theta(\tau) = (1 + 0.04 Ra_i^{\frac{1}{3}} \tau)^{-\frac{1}{3}} \ , \tag{2.12}$$

a relationship which is in good agreement with the experimental data. The large constant before τ shows that the conduction time-scale is much too long for this turbulently convecting system and the appropriate time scale is not t_c but $t_v = 25(d^2/\kappa)R_i^{-\frac{1}{3}}$ (see Table 2).

The development of the stagnant boundary layer at the base can be determined by solution of the conduction equation with appropriate boundary conditions. An approximate solution is given in Jaupart & Brandeis (1986). For our purposes it suffices to state that because it results from the solution of the diffusion equation, the thickness of the boundary layer δ scales with $(\kappa t_v)^{\frac{1}{2}}$, i.e. $\delta/d \sim R_i^{-\frac{1}{6}}$. Thus for a typical magma chamber a stagnant lower boundary layer may occupy of order 1% of the depth.

2.3. *Crystallization and compositional convection*

As a magma cools in a chamber it begins to solidify, preferentially at the colder boundaries, but also in the warmer interior. Magma is comprised of many chemical components, with those taken into the solid being almost always different from those in the neighbouring fluid. This generally means that the density of the depleted fluid close to the solidifying interface is different (either greater or less) than that of fluid nearby. This buoyancy difference (due to crystallization) leads to what is called *compositional con-*

vection, as described in greater detail in Chapter W by Worster. Because the density difference due to the compositional differences is generally very much larger than that due to the associated thermal differences, compositional convection is generally much more vigorous than, and dominates, any thermal convection present. The convection can mix the magma, carry small crystals along with it, and introduce strong chemical stratification, with almost non-interacting regions of magma, by the processes of double-diffusive convection (Huppert & Turner 1981a; Linden, Chapter L).

Exactly how the magma evolves depends on the geometry of the chamber and on the sign of the density difference of the released fluid. Even in the simplest case of cooling an initially uniform layer at a horizontal boundary leads to a 2x3 matrix of possibilities (Huppert & Worster 1985). The cooled boundary can be at the top of the layer, leading to an unstable thermal field, which is liable to thermal convection, or at the base of the layer, which results in a stable thermal field. In addition, the released fluid can be either positively or negatively buoyant or, as a somewhat special case, of equal density to the neighbouring fluid. Each of these six cases needs, and has been subject to, special investigation. The analyses include incorporation of the effects of mushy layers (chapter W) and thermodynamic non-equilibrium. In each case the quantitative predictions of the mathematical models have been in good agreement with the results of specially designed laboratory experiments using aqueous systems such as solutions of KNO_3, Na_2CO_3 or NH_4Cl (Huppert 1990, which contains numerous colour photographs of such experiments). Possibly the most surprising result, which has direct application to the interpretation of rock layers found at the base the frozen remains of some magma chambers, is that the cooling from above of a layer of fluid which releases less dense fluid on crystallization can lead to the evolution of a crystal layer on the floor of the container (Kerr *et al.* 1989). This is because undercooling in the interior of the fluid due to non-equilibrium effects at the crystallizing interface near the top of the layer (which must occur, at least to some extent, for crystallization to proceed) drives further crystallization at the base, as depicted in figure x.y of Worster.

If the cooling and crystallization takes place at a vertical side wall, the released fluid flows either up or down through the crystal mush, depending on the sign of its buoyancy, to form a separate layer at the top or bottom. With time, a strong vertical stratification can result, as described by Turner & Campbell (1986). The horizontal temperature gradient then couples with the vertical compositional gradient to lead to the inevitable double-diffusive layering (Linden, ch L), with vigorously convecting, almost uniform layers separated by thin interfaces across which there are (relatively) large changes in temperature and composition.

An interesting laboratory experiment, whose results are directly applicable to effects due to the sloping retaining walls of magma chambers, illustrate many of the above features. Huppert *et al.* (1986b) cooled an initially aqueous solution of Na_2CO_3 at an inclined plane, which was inserted into the container to divide the fluid into two geometrically identical halves. Upon crystallization on the upper surface of the inclined plane, the released, less dense fluid rose, mixed in with fluid of the upper region and induced a vertical stratification, which with time broke up into a series of double-diffusive layers. Crystallization on the lower surface of the plane also resulted in the release of less dense fluid, which could not rise because of the constraint of the impermeable plane above it. This fluid wound its way through the porous mushy layer to be deposited as a separate, ever growing layer at the top. As time proceeded the density of the released fluid decreased and hence displaced downwards fluid at the top of this layer by the 'filling box' mechanism (Linden p. ??). The resultant crystal shape on the two sides of the plane were quite different. The geometry of the two regions was identical, but the convective effects caused the solidification processes above and below the plane to be very different.

These general ideas on generating a stable stratification in magma chambers can be applied to understanding the interpretation of compositionally zoned volcanic products. In large magnitude explosive eruptions, tens to hundreds of km^3 of magma can be erupted in a few hours or days, often emptying a considerable proportion of the magma chamber. Such volcanic products are typically zoned in a systematic way in both chemical composition and inferred magma temperature. This indicates that the top of the chamber contained cool evolved magma, typically enriched in silica and volatiles, and the chamber was then stratified, with hotter and more silica poor magma occurring at depth (Sparks *et al.* , 1984). Such zoning is very common and can be satisfactorily explained by sidewall crystallization and compositional convection.

2.4. *Replenished chambers*

Replenishment of a magma chamber by new, hotter magma from below can revitalise the motions and initiate new processes. New magma can enter in batches in the form of solitary waves due to *compaction* deeper in the Earth, as described in §6 and Spiegelman (1993), but continuous 'seepage' of new magma may also be possible.

A particular situation, which illustrates how a new, and possibly counter-intuitive, process can arise in a replenished magma chamber, and indicates the dominant role of fluid mechanics, was considered theoretically by Huppert & Sparks (1980) and tested experimentally by Huppert & Turner (1981b). In the 1970's geologists had accumulated reliable evidence that the erupted output from many basaltic magma chambers was typically composed of approximately 50% silica and about 6 to 9% magnesium oxide (MgO). Different lines of argument suggested that the input magma had slightly less silica (c. 48%) but double or even triple the MgO content (about 12 to 18%). What is the reason for this large difference in MgO content?

Huppert & Sparks (1980) considered the illustrative case of a chamber filled with magma containing around 9% MgO at a temperature around 1250°C. New magma from deeper in the mantle, composed of 18% MgO at a temperature around 1400°C, is episodically intruded into the base of the chamber. This new magma, although hotter than the resident magma, is more dense owing to compositional differences, and hence forms a separate layer at the base. The bottom layer gradually cools because of its contact with the colder, upper layer, while remaining distinct because of the compositional difference. As it cools, the lower layer crystallizes to form small olivine crystals, which preferentially extract MgO from the melt, and thereby becomes less dense. The olivine is (mainly) kept in suspension in the melt by the vigorous convection in the lower layer. As the temperature difference between the layers approaches zero, the vigour of the convection eases, the relatively heavy olivine crystals fall to the base, the density difference across the double-diffusive interface vanishes and the MgO-depleted liquid rises into the main interior of the chamber, leaving behind a layer of compacted olivine crystals.

Huppert & Turner (1981b) modelled the essentials of this process in the laboratory, by feeding into a large reservoir a hot, heavy layer of aqueous KNO_3 beneath a colder, less dense layer of $NaNO_3$. As the lower layer cooled beyond the solidification point, crystals of KNO_3 grew in the presence of strong compositional convection in the lower layer. The density of the saturated aqueous KNO_3 decreased (at a rate in good agreement with theoretical calculations) until it reached that of the upper layer, at which point the liquid of the lower layer rose to mix with that of the upper – leaving behind the KNO_3 crystals.

These processes were consistent with evidence from old solidified magma chambers, such as on the island of Rum, off northwest Scotland, which displayed distinctive alternating layers of crystals representing solidification of magmas at alternatively high and low temperatures (and hence high and low MgO contents). There are numerous other geological examples of this process and the concept of a *density trap*, as it is sometimes referred to by Earth scientists, is frequently invoked in describing observations in the field.

The details of the injection of relatively heavy magma into a less dense ambient from either a point or line source to form a turbulent fountain, can be investigated using well-known concepts of turbulent plume theory (Turner, 1979 Ch. 6; Linden §??). When the Reynolds number of the input, Re_i, is very large and viscous effects can be neglected in both input and ambient fluid, of density ρ_i and ρ_a respectively, the height of rise of the fountain, h_f, is determined from the specific momentum and buoyancy fluxes at the source,

$$M_s = w_i^2 r_s^2 \qquad \text{and} \qquad B_s = w_i g_i' r_s^2 \tag{2.13a, b}$$

as

$$h_f = 2.5 M_s^{\frac{3}{4}} B_s^{-\frac{1}{2}} = 2.5 w_i (r_s/g_i')^{\frac{1}{2}} \ , \tag{2.14a, b}$$

where w_i is the input vertical velocity at the source of radius r_s of fluid with initial reduced gravity $g_i' = (\rho_i - \rho_a)g/\rho_a$. Campbell & Turner (1986) conducted a series of experiments, backed up by dimensional arguments, to investigate the influence of the viscosity of the input and ambient fluid, denoted by ν_i and ν_a respectively. They found that when $Re_i \geq 400$ mixing between input and ambient fluid is controlled exclusively by the Reynolds number of the ambient $Re_a = w_i r_s/\nu_a$ (and the value of Re_i has no significant influence). Mixing between the fluids, and the height of the fountain, both relative to the values at infinite ambient Reynolds number, is reduced by 10% (50%) as Re_a decreases below 70 (30).

For a magma chamber fed by a simple vertical conduit of radius r_m, the steady efflux can be calculated by balancing two quantities: the potential energy available to the magma of density ρ_m to rise because of its positive buoyancy with respect to the surrounding country rocks of density ρ_{cr} and the energy lost due to dissipative effects as the magma flows with mean velocity w along the rough sidewalls of the conduit. On the assumption that the flow in the conduit is turbulent this leads to an efflux rate

$$Q = \pi w r_m^2 = 2\pi (g'_{\rm cr}/f)^{\frac{1}{2}} r_m^{\frac{5}{2}} \ , \tag{2.15a, b}$$

where $g'_{\rm cr}$ is the reduced gravity of the surrounding country rock relative to the magma and f is a friction coefficient, which typically lies between 0.01 and 0.08. Combining (2.14b) and (2.15b), we obtain (within the Boussinesq approximation)

$$h_f = 5\left(\frac{\rho_{\rm cr} - \rho_m}{\rho_m - \rho_a}\right)^{\frac{1}{2}} \frac{r_m}{f^{\frac{1}{2}}} \ , \tag{2.16}$$

where $\rho_a < \rho_m < \rho_{\rm cr}$. With the insertion of typical values in (2.16), $h_f \sim 25 r_m$, which might vary between 25 m and 250 m. Thus for small conduits (and hence relatively small effluxes) effects due to the input fountain are also small. Only when the conduit radius (and associated Reynolds number) is quite large do fountains, and the associated mixing between input and ambient magmas, play an important role.

Input of magma less dense than the ambient occurs in some situations and for these cases the form of the flow of input magma has been investigated as a function of the Reynolds numbers of the input Re_i and ambient Re_a (Huppert *et al.* 1986). For sufficiently low values of Re_i and Re_a (each less than about 10), the input rises as a laminar conduit. With an increase in either Reynolds number the conduit becomes unstable and entrains ambient magma. For sufficiently large Reynolds numbers the input becomes turbulent and considerable entrainment can occur.

These ideas have been applied to understanding the observed mixing of basaltic magmas, the origin of some economic ore deposits, such as platinum sulphides (Campbell, Naldrett & Barnes, 1983) and the difficulty of erupting pure, unevolved mamgas.

2.5. *Melting*

Because temperature increases with depth within the Earth and relatively little heat is lost from a magma as it ascends, molten magma may enter a chamber at a temperature above the melting temperature of some of the components of the surrounding rock. Melting of the surrounding rock may then take place at the roof, base or sides of the chamber and the melting process will be strongly influenced by whether the melt is more or less dense than the ambient magma, or of identical density (which is rather unlikely). An important example occurs following the input of basaltic magma at around 1300°C into the granitic continental crust, which leads to a (granitic) melt that is about 0.4 g cm^{-3} less dense than the basaltic ambient.

As a fundamental example of melting, consider a heat flux H_i incident on a planar surface of a solid with melting temperature T_M (as sketched in figure 3a). Denoting the position of the surface at time t as $z = s$ and the temperature of the solid as $z \to \infty$ as T_∞, the temperature profile $\theta(z,t)$ in the solid and the rate of melting are determined from

$$\theta_t = \kappa\theta_{zz} \qquad (z > s) \tag{2.17}$$

$$\theta(z,t) = T_M \qquad (z = s) \qquad \text{and} \qquad \theta \to T_\infty \qquad (z \to \infty)\ . \tag{2.18a, b}$$

(on the assumption that the surface remains planar). A solution of (2.17) and (2.18), known sometimes as the ablation solution, valid away from $t = 0$, is given by

$$\theta(z,t) = T_\infty + (T_M - T_\infty)\exp[-V(z-s)/\kappa]\ , \tag{2.19}$$

where $V = ds/dt$ is the assumed constant melting rate. Equating the difference in heat fluxes across the interface to the rate of latent heat release (Worster X),

$$H_i - k\theta_z|_{z=s} = LV \ , \tag{2.20}$$

where L is the latent heat per unit mass of solid, and substituting (2.19) into (2.20), we determine that the melt rate

$$V = \frac{H_i}{\rho[L + c(T_M - T_\infty)]} \equiv \frac{H_i}{\mathcal{L}_*} \ . \tag{2.21a, b}$$

This relationship along with (2.19) represents a long-time solution; solutions satisfying particular initial conditions are discussed in Huppert (1989).

Consider now a semi-infinite, flat, solid roof, of melting temperature T_M, overlying a hot fluid, initially of depth D and at temperature T_0 ($> T_M$), the bottom of which, at $z = 0$, is thermally insulated (as sketched in figure 3b). If the density of the melt is greater than that of the original fluid and is miscible with it, the melt will mix with the fluid and the melting is determined entirely by thermal processes. The heat flux incident on the base of the melting roof is given by (c.f. 2.10)

$$H_i = 0.12k(\frac{\alpha g}{\kappa\nu})^{1/3}(T - T_M)^{4/3} \sim \Gamma \Delta T^{4/3} \ , \tag{2.22a, b}$$

where T is the temperature of the hot underlying fluid, $\Gamma \equiv 0.12k(\alpha g/\kappa\nu)^{1/3}$, $\Delta T = T_0 - T_M$ and (2.22b) is correct only initially, but gives a good indication of the scale of H_i for considerably longer. Combining (2.21) and (2.22), we obtain

$$\dot{s} = \Gamma \Delta T^{4/3}/\mathcal{L}_* \ , \tag{2.23}$$

which indicates that, at least initially,

$$s = (\Gamma \Delta T^{4/3}/\mathcal{L}_*)t \ . \tag{2.24}$$

The heat gained by the melt is taken from the fluid into which it mixes, which indicates that $\Delta T \delta s \sim -D\delta T$ and thus the nondimensional temperature of the lower layer

$$\Theta \equiv (T - T_0)/\Delta T \sim -s/D \propto -t \ , \qquad (2.25\mathrm{a,b})$$

i.e. Θ increases linearly with time. A more accurate quantification of this process is presented by Huppert & Sparks (1988a), who also describe laboratory experiments in which a wax roof was melted by a hot, aqueous salt solution to yield data on melt rates and fluid temperature which are in very good agreement with their theoretical predictions.

If the released melt is less dense than that of the hot fluid layer, it forms a separate layer beneath the roof, as sketched in figure 4. Initially heat is transferred by conduction across the melt layer as depicted in figure 4a. As the Rayleigh number of the layer increases, however, convection sets in. If the Rayleigh number becomes sufficiently large, as it does in natural situations, the convection becomes turbulent and the heat transfer can be analyzed using the four-thirds formulation. For illustrative purposes, we consider only the vigorously convecting state, as depicted in figure 4b. [The conductive state is analyzed fully by Huppert & Sparks (1988a).] We seek to determine the temperatures T_2 and T of the melt and fluid layer, respectively, the interfacial temperature T_1 and the thickness of the melt layer s as functions of time. For simplicity we shall assume here that the various phases all have the same values of physical parameters such as specific heat, thermal diffusivity, etc.

Because the upward heat flux from the hot fluid layer must equal that from the base of the melt layer

$$T_1 = \tfrac{1}{2}(T + T_2) \ . \qquad (2.26)$$

This heat flux flows primarily into the base of the roof and, under most circumstances, a very small portion of it is used to raise the temperature of the melt layer through which it passes. With the neglect of this effect

$$T_2 = \tfrac{1}{2}(T_1 + T_M) = \tfrac{2}{3}T_M + \tfrac{1}{3}T \ . \qquad (2.27\text{a, b})$$

Conservation of heat in the hot fluid layer requires

$$\rho c D \frac{dT}{dt} = -\Gamma(T - T_1)^{4/3} = -(\frac{1}{3})^{4/3}\Gamma(T - T_M)^{4/3} \ ,$$

which, subject to

$$T = T_0 \qquad (t = 0) \ , \qquad (2.29)$$

(where T_0 is the temperature of the layer when vigorous convection sets in at $t = 0$) has the solution

$$T = T_M + (\beta_1 + \beta_2 t)^{-3} \ , \qquad (2.30)$$

in terms of the two constants

$$\beta_1 = (T_0 - T_M)^{-1/3} \quad \text{and} \quad \beta_2 = \left(\frac{1}{3}\right)^{7/3}\Gamma/(\rho c D) \ . \qquad (2.31\text{a, b})$$

The ablation relationship (2.21) indicates that

$$\dot{s} = \Gamma(T_2 - T_M)^{4/3}/\mathcal{L}_* \ , \qquad (2.32)$$

which has the solution

$$s(t)/D = \rho c[\beta_1^{-3} - (\beta_1 + \beta_2 t)^{-3}]/\mathcal{L}_* \ . \qquad (2.33)$$

As $t \to \infty$, both T and $T_2 \to T_M$ while $s \to \rho c D(T_0 - T_M)/\mathcal{L}_*$. Expressed alternatively, after sufficient time has elapsed all the available heat in the hot fluid layer has been used to melt a specific finite thickness of the roof (and a negligibly small amount has been conducted away).

To make the model directly applicable to natural situations, Huppert & Sparks (1988a) evaluated the effect of allowing the lower fluid layer to crystallize and its viscosity to increase as the temperature falls. They used these results in two companion papers (Huppert & Sparks 1988b,c) which discuss the generation of granite by the melting of the continental crust due to the input of hot basaltic magma. The melting can be quite rapid, particularly if the crustal rocks are close to their melting temperature. The generation of large volumes of silicic magma, ready for eruption, can thus be accomplished in periods of decades when relatively hot crust is invaded by basaltic magma (rather than the tens of thousands of years previously conjectured by some geologists). The layer of rhyolite magma both grows and cools with time. The rhyolite magma must continually crystallize as it forms, with melting confined to the boundary of the system. The fundamental fluid dynamical concepts have also been applied quantitatively to the melting of ice sheets by basalt lava eruptions (Hoskuldsson & Sparks 1997).

Melting for the floor of a container has been considered by both Huppert & Sparks (1988c) and by Kerr (1994). Further research is needed on this topic to analyze the effects of melting a multi-component solid (cf. also the interesting comparisons between this section and section X.Y of Worster).

2.6. *Volatiles*

Dissolved gases or 'volatiles', particularly water and carbon dioxide, can play an important role in magmatic systems, both in magma chambers and, more centrally, in conduits feeding a volcanic eruption (as described in the next chapter). This is a consequence of the fact that smallish (but definitely non-zero) amounts of H_2O, CO_2, SO_2 and the like can be dissolved in magma. Due to a variety of processes, which include the most important ones of pressure release and crystallization, the magma can first become saturated and then release the dissolved volatiles as a gaseous phase (cf. opening, i.e. releasing the pressure in, a bottle of champagne). Because the density of gas is so much less than that of the liquid from which it originated (by approximately three orders of magnitude), the bulk density of gas plus liquid drops dramatically, or the pressure on the surrounding walls increases considerably, or, if the container is open, the volume increases rapidly (as in the champagne situation). All three of these can occur in various combinations.

Central to the argument is the solubility relationship, which can be expressed as

$$n_s = k_s\ p^{1/2}\ , \tag{2.34}$$

where n_s is the mass fraction of water, for example, in the magma at saturation, p the pressure and k_s a solubility constant, of the order 0.0014 bar$^{-1/2}$. For a system which crystallizes entirely anhydrous crystals, the dissolved water becomes driven into an ever decreasing liquid mass and the saturation level will decrease, as given quantitatively by $n_s = k_s(1 - X)\ p^{1/2}$, where X is the mass fraction of the crystalline phases. If the total mass fraction of water, N, exceeds n_s the difference will exsolve as gas, initially in the form of very small bubbles at nucleation sites. With the use of the perfect gas laws and the assumption of thermodynamic equilibrium, the bulk density ρ_b of gas plus liquid plus crystals can be expressed as

$$\rho_b^{-1} = RT(N - n_s)/P + (1 - N + n_s)/\rho\ , \tag{2.35}$$

where $\rho(P, T, X)$ is the density of the crystals plus melt, T is the temperature and R the universal gas constant. Values of ρ_b as a function of X for various values of N for $p = 1.5$ kbar ($\sim$ 500 m depth) are graphed in figure 5. For this value of p a magma with less than approximately 2.3% by weight of water crystallizes (to up to 50% by weight) without becoming saturated. For a range of higher water contents, at a specific crystallization level X_c (related directly to temperature), the magma becomes saturated. For $X > X_c$ water exsolves and the bulk density drops precipitously. Even for temperatures sufficiently high that no crystallization has yet taken place, it follows by extrapolation from figure 5 that a maximum of approximately 5.4% of water can be dissolved (at that pressure).

The replenishment of a magma chamber by relatively heavy, *wet* undersaturated magma can be investigated using these concepts (Huppert, Sparks & Turner 1982). Turbulent transfer of heat from the new lower layer to the upper layer leads to crystallization and exsolution of volatiles in the lower layer. As figure 5 shows, initial water contents of only a few per cent are sufficient for the bulk density of the lower layer to fall significantly, possibly to become equal to that of the upper layer and thereby lead to overturning and intimate mixing between the two different magmas. There are numerous petrographic observations of hybrid rocks associated with volcanic eruptions worldwide which can be explained by such a mechanism of mixing. For example, one of the most common volcanic rock types on Earth is formed at island arcs where plates collide. The magma is called *andesite* and is intermediate in composition between basalt and rhyolite. Commonly, andesites are mixtures of basalt and rhyolite. The mechanism just described provides a possible explanation for this mixing.

The fundamental ideas and the subsequent eruption which such mixing can trigger was demonstrated experimentally by Turner, Huppert & Sparks (1983), who repeated the experiment performed by Huppert & Turner (1981b) and described in §2.4, but added a small amount of HNO_3 to the lower layer and some $CaCO_3$ to the upper layer. These chemicals were held separate by double-diffusive effects operating in the two layers until the interface between them broke down. Subsequently, the acid and base produced CO_2 which bubbled vigorously through the system and frothed out of the simple volcanic crater made of perspex which Turner *et al* had placed over the top of the container.

3. The propagation of magma through the crust

An increase in pressure in a magma chamber beneath a volcano, due either to local processes, such as discussed at the end of the last section, or to large-scale tectonic movement, can cause the magma to rise through the Earth's crust and erupt at the surface. The magma flows in conduits or dykes in a manner controlled by elastic, fluid dynamic and thermodynamic effects. Basaltic magmas, such as those in Hawaii, in Iceland and along the mid-ocean ridges, are relatively low in both volatile content and viscosity and produce non-explosive eruptions of comparatively dry magma. The dry magma often comes to the surface through long (between 1 and 10 km), relatively straight fissures, which are of order one metre in width and may extend from between 10 to 100 km in the other horizontal direction (figure 6). Silicic magmas, on the other hand, such as those erupted on the continents and from descending lithospheric plates, tend to have relatively high volatile content and viscosity. They hence tend to produce more explosive eruptions, such as occurred at the Soufriere Hills volcano on Montserrat, Mt Pinatubo in the Phillipines, Redoubt in Alaska and Unzen in Japan. The magma conduits from chamber to surface tend to be of roughly circular cross-section and the erupted material is a complicated mixture of solid ash particles, liquid magma and gas.

We commence our description of the flow of magma through the crust by considering effects induced in dry magmas and then discuss the extra effects accompanying the transport of wet magmas, which can exhibit considerable pressure exsolution as they rise.

Any fluid flowing in a laminar manner due to a (local, negative) pressure gradient ∇p in either a channel of slowly varying width or a pipe of local radius a has velocity profile

$$u = -\gamma_u (\nabla p/\mu)(a^2 - \zeta^2) \ , \tag{3.1}$$

where ζ is either: the co-ordinate across the channel (with the walls at $\zeta = \pm a$) in which case $\gamma_u = \frac{1}{2}$; or the radial co-ordinate, in which case $\gamma_u = \frac{1}{4}$. The associated volume flux Q, is given by $Q = -\gamma_Q \nabla p/\mu$, where $\gamma_Q = a^3/3$ for a two-dimensional channel and $\gamma_Q = \pi a^4/8$ for circular geometry.

If, on the other hand, the Reynolds number of the flow based on the radius a exceeds a value of order 1000, the flow is turbulent, the mean velocity profile is effectively uniform and the volume flux is given in terms of a friction coefficient f by

$$Q = 2\pi(-\nabla p/\rho f)^{1/2}/a^{5/2} \ . \tag{3.2}$$

3.1. *Two-dimensional fissure eruptions: thermodynamics and fluid mechanics*

This subsection concentrates on two ingredients of magma flow in dykes: the fluid mechanics of flow along an already open dyke; and the heat transfer from the dyke to the surrounding country rock, which can result in either solidification of the magma or melting of the country rock. Elastic effects in the surrounding rock, in particular the mechanisms by which a fluid dyke is initiated in solid rock, will be discussed in the next subsection.

The starting point of the model is a long, two-dimensional dyke, of initially uniform width W_i through which relatively hot, Newtonian fluid of uniform viscosity μ ($\sim 100 Pa\ s$) is driven by a pressure drop, ΔP. Heat is transferred by both advection and conduction. Because the width of a dyke $W(\sim 1m)$ is so much less than its length L ($\sim 1km$), conduction along the dyke can be neglected (with respect to conduction across the dyke). In the surrounding rock, heat can only be transferred by conduction, as determined by the two fixed temperatures of T_∞ (typically between 0 and 1000°C) in the far field and T_W ($\sim$ 1150°C) at the dyke wall. With flow rates $\overline{u}$ of order $1ms^{-1}$, the Reynolds number (of order 10) is sufficiently low for the flow to be laminar.

The time for the magma in the main dyke flow to traverse the length L is $L/\overline{u}$ ($\sim$ 20 min), in which time a thermal boundary layer of width $\delta = (\kappa L/\overline{u})^{1/2} \sim 3cm \ll W_i$ is formed, where the thermal diffusivity in the magma, $\kappa \sim 10^{-2}cm^2s^{-1}$. Within the boundary layer the main Poiseuille flow appears as a uniform shear flow, of strength γ_s ($\sim 1s^{-1}$). Each of W, δ and γ will vary gradually along the dyke and slowly with time.

Consider a locally Cartesian coordinate system, moving horizontally with the dyke wall, which employs a vertical z axis, with $z = 0$ at the base of the dyke, and a horizontal y axis, with $y = 0$ at the dyke wall, so that $y > 0$ in the magma and $y < 0$ in the surrounding solid. The wall will migrate with velocity v due either to solidification of magma against the wall ($v > 0$) or to melting of the solid by the flow ($v < 0$). With identical values of the thermal diffusivities in magma and solid, the initial-value problem for the temperature in both fluid and solid $T(y, z, t)$ can then be stated, using the usual boundary-layer assumptions, as

$$T_t - vT_y + \gamma_s y T_z = \kappa T_{yy} \qquad (y, t > 0) \tag{3.3}$$

$$T_t - vT_y = \kappa T_{yy} \qquad (y < 0, t > 0) \tag{3.4}$$

along with boundary and initial conditions

$$T = T_W \quad (y = 0) \qquad T \to T_0 \quad (\text{either } z \text{ or } t = 0, y > 0, \text{ and } y \to \infty)$$

$$T \to T_\infty \quad (t = 0, y < 0 \text{ and } y \to -\infty) \ , \tag{3.5a, b, c}$$

where T_0 ($\sim 1200°C$) is the temperature of the magma at the base of the dyke.

The velocity of migration of the interface between liquid and solid is proportional to the difference in the conductive fluxes across the wall, as expressed by (cf. Worster)

$$(L/c)v = \kappa[T_y(0-, z, t) - T_y(0+, z, t)] \ , \tag{3.6}$$

where L is the latent heat per unit mass of either melting or solidification and c the specific heat per unit mass. With the use of standard relationships for Poiseuille flow, the volume flow rate $Q(t)$ and local dyke width $W(z, t)$ are related to ΔP and γ_s through

$$\Delta P = 12\mu Q(t) \int_0^L W^{-3}(z, t) dz \qquad \text{and} \qquad \gamma_s(z, t) = 6Q(t)W^{-2}(z, t) \ . \tag{3.7a, b}$$

Owing to either solidification or melting at the walls, the width of the dyke gradually changes according to

$$W(z,t) = W_i - 2\int_0^t v(z,t')dt' \ . \tag{3.8}$$

The solutions of (3.3)-(3.8) depend on the Stefan number of the magma $S_0 = L/[c(T_0 - T_W)] \sim 20$ and that of the solid $S_\infty = L/[c(T_W - T_\infty]$, which typically takes on values between 1 and 10 dependent on the far-field temperature of the country rock due to previous dyking episodes. At first both fluid and solid temperatures adjust to T_W, during which time fluid *must* solidify at the wall because of the very large (initially infinite) conductive heat flux into the wall [cf. the initial-value response of hot fluid running over a cold floor as discussed by Huppert (1989)]. Subsequently the continual supply of hot fluid transfers heat into the wall, which may melt it, while the solid continues to adjust to T_W.

Considerable analysis of the system (3)–(8) has been carried out (Bruce & Huppert 1989; Lister & Dellar, 1996), which when complemented by numerical integration leads to the following results. For dykes less than a critical width (dependent on the input parameters) the initial solidification at the walls continues until the dyke becomes blocked, which first occurs at the surface, and the eruption ceases. For dykes originally broader than this critical width, before solidification can close the conduit, the advected heat flux melts back the newly solidified magma and melting occurs along the entire length of the dyke. The width then gradually increases (as long as the driving pressure is maintained). These conclusions are summarized quantitatively in figure 7.

Such analyses of the fluid dynamics and heat transfer in dykes can help explain many aspects of dyke geology and basalt lava eruption. Typically, dykes are observed from seismic data to propagate at speeds of 0.1 to 1 m s^{-1}. Such speeds are broadly consistent with the flow conditions expected along dykes of width 0.5 to 2 m. The strong sensitivity of flow and heat transfer to dyke width can also help explain the localisation of flow in basalt fissure eruptions. As solidification constricts the narrower part of the dyke and chokes off flow, melting can widen the widest parts, eventually focussing flow locally. In long-lived eruptions, melting in focussed regions can lead to formation of cylindrical conduits.

3.2. *Fluid mechanics and elasticity*

In reality the magma needs to force its way through the solid surrounding rocks: dykes are not supplied, but need to be made. This brings into consideration elastic responses in the rock. A series of solutions incorporating effects of both fluid mechanics and elasticity are now developed. The conclusions are that with values of natural physical parameters the flow is mainly characterized by a local balance between buoyancy and viscous forces; elastic forces play a secondary role except near the tip of the dyke.

Consider the laminar flow of fluid through a channel of width $W(x,z,t)$ with respect to horizontal and vertical coordinates x and z. Local continuity requires that

$$\frac{\partial W}{\partial t} = -\nabla \cdot \mathbf{Q} = \frac{1}{12\mu} \quad \nabla \cdot (W^3 \boldsymbol{\nabla} p) \ . \tag{3.9a, b}$$

If the dyke is two-dimensional — its width independent of x — the theory of linear elasticity indicates that the pressure p_e on the fluid due to elastic deformation of the surrounding solid (assumed to be of infinite extent) is given by

$$p_e = -m\boldsymbol{\mathcal{H}}\left\{\frac{\partial W}{\partial z}\right\} \equiv -\frac{m}{\pi}\int_{-\infty}^{\infty} \frac{\partial W(s,t)}{\partial s}\frac{ds}{s-z} \ , \tag{3.10a, b}$$

where $\boldsymbol{\mathcal{H}}\{f\}$ is the Hilbert transform (Miles 1971) of $f(z,t)$ with respect to z and m is a material constant of the rock $\sim 30-50\ GPa$ for basalts and $10-20\ GPa$ for granites. The magma forces its way through the solid by the residual pressure due to buoyancy, $p_b = -\Delta\rho g z$, which arises because the magma density ρ_m is less than that of the solid ρ_s by a (positive) amount $\Delta\rho \sim 300 kg\ m^{-3}$ for granite flowing through basalt. The pressures due to buoyancy and elasticity add linearly to yield the total pressure p driving the flow. Substituting this result into (3.9), Spence, Sharp & Turcotte (1987) obtained the governing integro-differential equation

$$\frac{\partial W}{\partial t} + \frac{1}{12}\Delta\rho\frac{g}{\mu}\frac{\partial W^3}{\partial z} = -\frac{m}{12\mu}\frac{\partial}{\partial z}\left[W^3\frac{\partial}{\partial z}\boldsymbol{\mathcal{H}}\left\{\frac{\partial W}{\partial z}\right\}\right]. \tag{3.11}$$

The two terms on the left represent fluid mechanical effects; the one on the right elastic effects.

Suppose that the dyke is fed by a line source which releases a constant flux q per unit length. The dyke will then rise at a constant speed, say c, which can be found by determining a travelling wave solution of the form $W(z,t) = W(\eta \equiv ct - z)$. Far from the front of the dyke W will tend to a constant value, say W_∞, where the elastic term on the right of (3.11) is dominated by the fluid mechanical terms on the left because of the smaller number of derivatives involved. In this limit $\partial p/\partial z = -\Delta\rho g$ and from (3.9b) $W_\infty = (12\mu q/g\Delta\rho)^{1/3}$ (independent of the value of the elastic constant m). Substituting the postulated wave form of W into the left-hand side of (3.11), integrating the result and evaluating the resulting constant by realizing that ahead of the dyke its width is zero, we determine that $c = q/W_\infty$ (as it must from continuity) and $dp/d\eta = 12\mu c/W^2$.

For small distances ζ in the vicinity of the crack tip there must be a singularity of the form $p \sim -\Lambda\zeta^{-1/2}$ where Λ is a material constant known as the stress-intensity factor $\sim 6\times 10^6\ Pa\ m^{1/2}$. A weaker singularity at the front does not allow the crack to propagate, while a stronger singularity would suggest it propagates far too fast.

With the inversion of the Hilbert transform (3.10), use of $dp/d\eta = 12\mu c/W^2$ and some algebraic manipulations, the problem can be expressed as a nonlinear integral equation for $W(\eta)$ subject to a side condition incorporating a non-dimensional stress intensity factor $\Lambda^* = \Lambda/\left(\frac{3}{8}m^3\mu q\right)^{1/4} \sim 10^{-2}$ for natural values of the parameters. This small value suggests that the singularity at the front plays a negligible role in the shape (or propagation) of the dyke. The solution to this problem must be obtained numerically

(Lister 1990a); and the resulting dyke profile for $\Lambda^* = 0$ (and 1 for comparison) is presented in figure 8. The front of the propagating crack is bulbous with a maximum width of $1.3W_\infty$. Elastic effects play a negligible role behind the bulbous nose whose length, proportional to $(mW_\infty/g\Delta\rho)^{1/2}$, is of order 1 km.

The release of a flux Q from a *point* (in contrast to a line) source can be analyzed using similar concepts. With respect to perpendicular horizontal axes x and y and a vertical axis z with the origin of co-ordinates at the source, consider a planar, steady-state dyke described by $|y| < \frac{1}{2}W(x,y)$ for $|x| < B(z,t)$. After the dyke has propagated some distance, $W \ll B \ll h$, where h is a representative height, and a similarity solution can be found (Lister 1990b) of the form

$$B(z) = \beta\, z^{3/10} \text{ and } W(x,z) = \gamma\, z^{-1/10}(1-\zeta^2)^{3/2}\ , \quad \text{where} \quad \zeta = \beta^{-1}xz^{3/10}\ , \quad (3.12)$$

$\beta = 2.6(Q\mu m^3)^{1/10}/(g\Delta\rho)^{2/5}$ and $\gamma = 0.18[Q^3\mu^3/m(g\Delta\rho)^2]^{1/10}$. For a range of Q between 1 and $10^6 m^3 s^{-1}$ and with representative values of the physical parameters, W typically ranges between a few centimetres and a few metres, while B ranges between a few kilometres and several tens of kilometres. These values are consistent with observations of dykes, whose widths range from between 0.1 to 10 m, with the most common widths being between 0.5 and 2 m. It should be noted, however, that over a considerable portion at the lower ends of these ranges, which reflect the lower values of Q, solidification at the edges and thermal erosion near the centre would play an essential, and as yet unincorporated, role.

Throughout this subsection the densities of the magma and the surrounding solid have been considered constant. In reality the density of solid rock tends to increase with depth. One can thus envisage magma propagating upwards through the lithosphere as a result of its (decreasing) excess buoyancy until the density of the magma equals that of the surroundings, whereupon the magma will intrude laterally at what has been called the level of neutral buoyancy (LNB). This is similar to the lateral intrusion of fluid at the top of a plume in a stratified environment, as described by Linden (p. X) and in §4.3. Elegant similarity solutions for a fluid-filled crack propagating at its LNB, either at an interface between two semi-infinite solid layers of different densities or into a density stratified solid medium are reviewed by Lister & Kerr (1991). Field observations indicate, however, that the density of some magmas which are considered to have intruded laterally into the surrounding rock is quite different from that of the surrounding solid medium. This subject may reflect a typical little cameo of geophysics: an attractive theory suggests that observationalists re-examine their findings, which leads to the theory being found deficient, at least in some of its applications, until a more complete (and maybe less elegant) series of processes is incorporated.

3.3. *The ascent of wet magmas*

As discussed in §2.6 and at the beginning of this section, the release of volatiles can play a large role in magma dynamics, especially in relatively viscous, silicic magmas. Owing to the exsolution, such magmatic systems generate the most explosive eruptions. The bubbles that form range in size from 10^{-4} to 1 cm, which, due to their buoyancy, would rise relative to the surrounding viscous magma at speeds between 10^{-15} and $10^{-6} ms^{-1}$. This is so slow that it is thus generally assumed that the bubbles travel with the magma and form a homogeneous mixture. For every 1% by weight of volatiles exsolved, the viscosity of the mixture *increases* by an order of magnitude until, at volatile contents of about 72% by volume, the mixture behaves more like a foam than a simple Newtonian liquid incorporating numerous bubbles, as reviewed by Woods (1995).

The steady eruption at mean speed u of magma of density ρ from a chamber along a vertical conduit of prescribed shape can be described by the one-dimensional equations

$$\rho u A = Q \ , \tag{3.13}$$

which represents the conservation of mass flux Q in a conduit of cross-sectional area A, and

$$\rho u \frac{du}{dz} = -\frac{dp}{dz} - \rho g - f \ , \tag{3.14}$$

which represents conservation of momentum, where z is a vertical co-ordinate and f denotes the frictional dissipation. The motion of bubbly liquids is still not sufficiently well understood to determine f completely. A simple approximation, while the gas content is reasonably small and the flow laminar, would be to set $f = 8\pi\mu u/A$, as appropriate for a Newtonian viscous liquid. This relationship has been augmented by resort to empirical functions which incorporate effects of the total weight fraction of water N and the fraction by volume of gas, or void fraction, ϕ. As ϕ increases the magma becomes increasingly foamy until, at a void fraction of approximately 75%, the gaseous magma fragments and changes from a bubbly liquid to an ash- and liquid-laden gas, with greatly reduced viscosity. Thereafter, frictional effects are sufficiently small that f can be ignored.

A relationship between pressure, density and volatile content can be formulated as explained in §2.6 to lead to

$$\rho^{-1} = RT(N - n_s)/p + (1 - N + n_s)/\rho_l \ , \tag{3.15}$$

where ρ_l is the density of the liquid. This relationship can be rearranged to indicate that

$$\phi^{-1} = 1 + (1 - N + n_s)p_s/[(N - n_s)RT\rho_l] \ . \tag{3.16}$$

Differentiating (3.13) with respect to z after taking the logarithm of both sides and (3.15) with respect to ρ on the assumption that the flow in the conduit is sufficiently rapid that T remains effectively constant, we can write (3.14) as

$$\frac{dp}{dz} = [(\frac{\rho u^2}{A})\frac{dA}{dz} - \rho g - f]/(1 - \frac{u^2}{a_s^2}) \ , \tag{3.17}$$

where $a_s^2 = dp/d\rho$, the square of the speed of sound in the bubbly magma, is given by

$$a_s^2 = (p/\rho)^2[(N - \frac{1}{2}n_s)RT - (p/\rho_l)]^{-1} \ . \tag{3.18}$$

The speed of sound in this multi-phase mixture is generally considerably less than that in a pure gas because the mixture displays the compressibility of a gas with the much higher density, and hence inertia, of a solid or liquid. Equation (3.17), along with the subsidiary equations (2.34), (3.13) and (3.18), can be integrated numerically starting from a given pressure excess (typically between 0 and 100 bar) at the base of the conduit and an assumption of an empirical relationship for the viscosity of the system as a function of ϕ and n_s. So far this has only been done for a straight-walled chamber, $dA/dz \equiv 0$, along with the assumption that the flow does not become supersonic within the conduit. It is well known from the field of gas dynamics that flows become supersonic at either a flow constriction ($dA/dz < 0$) or at an open end. Real magmatic conduits no doubt have numerous constrictions and so many interesting results are still to be found. Typical solutions, assuming that the flow is sonic at the vent, 3 km above the base, are graphed in figure 9.

As the magma rises and decompresses, ϕ and u increase and, up to the fragmentation level, the viscosity of the magma increases. Beyond the fragmentation level the flow is effectively inviscid and so the resulting pressure gradient is very much less and virtually constant. The flow exits at a pressure considerably in excess of the atmospheric pressure (typically between 10 and 50 times greater) with $u = a_s$. Numerical evaluation of (3.18) indicates that for $0.03 < N < 0.07$ and 1 bar $< p <$ 100 bar, a_s does not depart by more than 5% from $0.93(NRT)^{1/2}$, which gives a way of estimating the exit speeds without detailed solution of the governing equations. For example, for $T = 10^3 K$, $a_s = 115(200) ms^{-1}$ for $N = 0.03(0.06)$. This small variation in exit speed, coupled with the small change in magma density, indicates (through 3.13) that the mass eruption rate is primarily related to the cross-sectional area of the vent, A. Because the exit flow is heavily overpressured a significant decompression phase must be experienced close to the vent. This is really part of the eruption column dynamics, which is treated in the next chapter.

Finally, extra effects, not yet fully understood, can arise due to crystallization, supersaturation or the delayed nucleation of exsolved bubbles: typical kinetic and disequilibrium effects. Preliminary experiments suggest that supersaturations corresponding to over 100MPa may be feasible. This would constrain the presence of bubbles until just below the fragmentation level and alter the details of the numerical calculations considerably. Some discussion of these effects is presented in chapter 3 of Sparks *et al.* (1997).

4. Fluid mechanics and thermodynamics of volcanic eruption columns

Once out of the volcanic conduit, the explosive mixture of gas and ash intrudes into the atmosphere and reacts with it. Recent eruption columns have risen as much as 45 km before invading laterally into the atmosphere to cause local, and even global, changes in the weather and climate. This section presents the main fluid-mechanical concepts used to describe eruption columns, explains why the initially heavier-than-air explosive mixture can penetrate so far into the atmosphere and obtains quantitative relationships for the properties of volcanic eruption columns.

4.1. *The decompression phase*

The turbulent multiphase eruption jet is modelled to exit from the top of the vent at sonic speed and exit pressure p_e considerably higher than the atmospheric pressure p_a.

A decompression phase must then follow. Consider the vent to open directly into the atmosphere, rather than through a large expanding crater, a situation which will be considered explicitly later. The decompression takes place, as sketched in figure 10, over a relatively short distance (less than a few hundred metres) during which the flow can be considered steady and effects of gravity, friction and fluid entrainment neglected. (This is **not** appropriate higher up in the eruption column)

The governing equations (Woods 1995) are then mass continuity, expressed by (3.13), momentum continuity

$$\rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p \tag{4.1}$$

and conservation of enthalpy

$$\rho \mathbf{u} \cdot \boldsymbol{\nabla}[c_v T + (p/\rho) + u^2/2] = 0 \ , \tag{4.2}$$

where c_v is the specific heat at constant volume. Integrating (4.1) over the control volume $\mathcal{V}$ with the use of (3.13), we determine the velocity at the end of the decompression phase u_0, when the pressure has fallen to p_0, as

$$u_0 = u_e + A_e(p_e - p_0)/Q \approx u_e + A_e p_e/Q = u_e + p_e/(\rho_e u_e) \ , \tag{4.3a, b, c}$$

where the subscript e denotes values at the exit point at the top of the vent (and $u_e = a_s$).

Numerical examination of the terms within the square brackets of (4.2) indicates that the specific heat $c_v T$ is of order $10^6 \ Jkg^{-1}$, while each of the other two terms is very much less and of order $10^4 \ Jkg^{-1}$. Thus the temperature is virtually constant across the decompression region and can be equated to that at the vent, which in turn is virtually identical to that of the original magma in the chamber $T \sim 1000°C$.

At the relatively low pressure of the atmosphere, (2.4) indicates that most of the originally dissolved volatiles have been exsolved by the end of the decompression phase ($n_s \ll N$) and the volume of the gas phase greatly exceeds that of the other phases.

Thus the second term on the right of (3.15) is negligible compared to the first term and the density of the flow at the end of the decompression phase $\rho_0 \simeq p_0/(NRT)$. Using the form of this relationship to describe *very approximately* the exit conditions, we substitute $p_e \approx \rho_e/(NRT)$ into (4.3) to determine that $u_0 \simeq 1.9(NRT)^{1/2}$. An accurate numerical evaluation shows that for $0.02 < N < 0.07$ and $p_0 < p_e/10$, $1.7 < u_0/(NRT)^{1/2} < 1.9$, with velocities falling below the lower value for relatively low exit pressures, which correspond to low values of eruption rates Q or vent radii. The area of the jet at the end of the decompression stage, that is at the effective base of the eruption column, A_0, is hence given by $A_0 = Q/(u_e\rho_0) \simeq 0.5Qp_0(NRT)^{1/2}$.

If the eruption conduit opens into a large crater, additional phenomena are possible. The sonic flow at the vent will become supersonic within the diverging walls of the crater and the term in dA/dz dominates (3.17). The pressure decreases and, analogous to classical gasdynamic flows from nozzles (Liepmann & Roshko, 1957), three different types of flow can issue from the top of the crater. Firstly, the pressure may remain above atmospheric and an overpressured, supersonic flow results. At lower exit pressures the flow can decompress relatively further in the crater and an underpressured supersonic flow results. In both these situations a short adjustment region above the crater is required to bring the flow to atmospheric pressure. At even lower exit pressures, a shock develops within the conduit and the material leaves the crater subsonically at atmospheric pressure.

4.2. *Eruption columns*

Vertical exit velocities of a few hundred metres per second for a heavily ash-laden and gas-dominated flow are impressive, but are nowhere near sufficiently large to take the eruption column many tens of kilometres into the atmosphere, as is observed, merely by converting the available kinetic energy into potential energy. (At the upper range, $u_e = 400 m\ s^{-1}$, $u_e^2/2g \simeq 8\ km$, almost six times too small.) Eruption columns penetrate deep into the stratosphere by converting *thermal* energy into potential energy in a way that will now be described.

The mass fraction of small solids – the volcanic ash – is typically between 0.95 and 0.98, although the volume fraction may be only of order 10^{-4}. The large mass fraction makes the density of the eruption column at its base typically between 20 and 50 times that of the surrounding atmosphere. As the flow in the eruption column develops, the plume entrains the relatively cold surrounding air (as described by Linden Ch. X.Y). The small, hot ash particles readily transfer their heat to the engulfed air and so the bulk density of the plume decreases with height (because the heated air is less dense), while at the same time its upward velocity decreases, partially due to gravitational effects operating on the relatively heavy material and partially due to the necessity of imparting upward momentum to the initially stationary entrained air. The competition between the decreasing density and velocity lead to the two fundamental styles of volcanic eruption. Either the decreasing density dominates and the material of the column becomes relatively buoyant, to lead to what is know as a Plinian eruption column, examples of which include the eruptions of Vesuvius in AD79 documented by Pliny, St Helens in 1980 and El Cichon in 1982; or the upward velocity ceases before the material becomes buoyant and a collapsed fountain develops which results in a ground-hugging, ash-laden surge known as a pyroclastic flow, such as occurred at Taupo in AD186, Ngauruhoe in 1973 and Pinatubo in 1991. A full description of these flows will be delayed until the next section.

The equations of motion in the entraining plume, which rises through an atmosphere of spatially-varying properties, are based on the pioneering entrainment assumption of Morton, Taylor & Turner (1956) as described by Linden (ch. X.y) and Turner (1979, 1986). Allowing for the considerable changes in density involved, i.e. **not** making the

Boussinesq approximation, we write the conservation of mass and momentum equations with respect to the vertical z-axis in standard form as

$$\frac{d}{dz}(\rho b^2 u) = 2\alpha\rho_a(z)bu \qquad \text{and} \qquad \frac{d}{dz}(\rho b^2 u^2) = \rho_a b^2 g', \tag{4.4a, b}$$

where $\rho(z)$ is the mean density in the column of radius $b(z)$, in which the fluid is propagating with mean vertical velocity $u(z)$ through an atmosphere specified by its density $\rho_a(z)$, α is the entrainment constant $\simeq 0.1$ and $g'(z) = (\rho_a - \rho)g/\rho_a$. To (4.4) must be added the energy equation (Woods, 1995)

$$\frac{d}{dz}\left[\rho b^2\left(c_p T + \frac{1}{2}u^2 + gz\right)u\right] = 2\alpha\rho_a(z)bu(c_p T_a + gz) \ , \tag{4.5}$$

where c_p is the specific heat at constant pressure, $T(z)$ and $T_a(z)$ are the temperatures within the plume and surrounding atmosphere respectively, with $T_a(z), \rho_a(z)$ and $p_a(z)$ linked by

$$p_a(z) = \rho_a(z)RT_a(z) = p_0 - g\int_0^z \rho_a(z')dz' \ . \tag{4.6a, b}$$

The ordinary differential system (4.4)–(4.6) needs to be integrated numerically from $z = 0$ given initial values of the mass flux Q, momentum flux Qu_0, energy flux $\mathcal{E}$ and a functional form for one of $\rho_a(z), p_a(z)$ or $T_a(z)$. The "standard" calculation assumes an initially decreasing temperature profile in the atmosphere with constant lapse rate $6.5K\ km^{-1}$ up to a tropopause of between 8 and 17 km, followed by a constant temperature regime up to the base of the stratosphere at 21 km, with a temperature profile increasing at $2K\ km^{-1}$ beyond that.

A typical set of results is shown in figure 11, which presents the velocity and density deficiency in the jet for three different values of u_0. For the largest of these ($200m\ s^{-1}$) the eruption column density falls off rapidly due to entrainment and the material in the column becomes buoyant at a height of about 200 m. For the smallest u_0 ($50m\ s^{-1}$) the column runs out of upward momentum (coincidentally at about the same height) and a collapsing fountain occurs. Figure 12 presents the broad curve that separates Plinian behaviour from the occurrence of a collapsed fountain. A Plinian column is favoured by lower mass eruption rates and higher eruption velocities.

The original theory of Morton *et al.* (1956) calculated the height of rise H of a buoyant plume in a stratified environment to be given by

$$H = 5F_0^{1/4}N^{-3/4} \ , \tag{4.7}$$

where F_0 is the initial specific buoyancy flux, here given by $F_0 = Q(T - T_0)g/(\rho_0 T_0)$, and N is the (assumed constant) buoyancy frequency of the atmosphere. Figure 13 presents the theoretical relationship (4.7) using standard atmospheric values which leads to $H = 260Q^{1/4}m$ if Q is expressed in $kg\ s^{-1}$. For comparison, the figure plots the maximum penetration heights of 12 volcanic eruptions this century. The agreement is remarkably good; and for the 45 km high Bezymianny eruption represents an extrapolation (for the use of the value of entrainment coefficient $\alpha \simeq 0.1$) over more than five orders of magnitude from the 20 cm measurements in the laboratory.

Other physical effects play an additional, albeit secondary, role in real eruption columns. The main ones are moisture in the atmosphere and particle fall out in the plume. The former requires incorporation of standard *wet* thermodynamics in the atmospheric modelling, as laid out in Sparks *et al.* (1997). A discussion of the latter is deferred until the next section. Note that (horizontal) winds in the atmosphere tend to play a rather small role on the column itself, because the (vertical) velocities and turbulent intensities in the plume are so large. Generally, the scale of the vertical velocity w_s takes on values between 40 and 400 ms^{-1}. If the typical *horizontal* wind velocity U is very much less than w_s it has very little influence on the plume. If $w_s \ll U$ a weak, bent-over plume develops and the eruption penetrates far less into the atmosphere. Wind may also be important, however, in the dispersion of the final intrusion, known as an umbrella cloud, as will now be described.

4.3. *The development of an umbrella cloud*

The mushroom shaped top of an eruption column, which slowly intrudes laterally at its own density level into the stratified atmosphere, is one of the more awesome sights of a volcanic eruption (Figure 14). The thickness of the umbrella cloud, h, determined from observations on volcanic eruptions appears to be roughly $\frac{1}{4}H$, with approximately half the cloud above H and half below that height.

If there is no wind, the cloud intrudes radially, driven by the horizontal pressure gradient which arises because of the different vertical hydrostatic pressure gradients in the cloud of mean density $\overline{\rho}$ and the atmosphere. Modelling the cloud as an expanding disc or cylinder of thickness $h(t)$ and radius $R(t)$ and neglecting both entrainment of the ambient and particle fall-out, effects which will be discussed in the next section, we can write the mass conservation equation as

$$\pi\overline{\rho}\frac{d}{dt}(R^2h) = Q_H \ , \tag{4.8}$$

where Q_H is the mass flux of the intruding cloud (which is very much larger than that at the base of the column because of entrainment). Integrating (4.8) on the assumption that the umbrella cloud already has a radius R_0 at the start of the lateral intrusion at $t = 0$, we find that the volume of the cloud, $\pi R^2 h$, increases linearly with time as

$$\pi R^2 h = (Q_H/\overline{\rho})t + V_0 \ , \tag{4.9}$$

where $V_0 = \pi R_0^2 \ h_0$ is the initial volume of the cloud.

The cloud is an example of a gravity current, as discussed in much more detail in the next section, which, on the large scale considered here, propagates under a balance between inertial and buoyancy forces (and frictional effects are negligibly small). By dimensional analysis, or by use of Bernouilli's theorem, it can be shown that the horizontal momentum equation can be replaced by the Froude condition

$$\frac{dR}{dt} = Fr(g'h)^{1/2} \ , \tag{4.10}$$

where the reduced gravity $g' = (\overline{\rho} - \rho_H)g/\rho_H$, ρ_H is the density of the atmosphere at height H and the Froude number Fr is constant $\simeq 1.19$ (§5.2). In the vicinity of H the stratification in the atmosphere is effectively linear and so $g' \simeq N_H^2 h$, where N_H is the buoyancy frequency of the atmosphere at a height H. Substituting this relationship into (4.10), we find that $h = dR/dt/(FrN_H)$, which upon substitution into (4.9) and integration yields

$$R^3(t) = \frac{3Fr}{2\pi} N_H \left(\frac{Q_H}{\overline{\rho}} \right) t^2 + \frac{3Fr}{\pi} N_H V_0 t + R_0^3 \ . \tag{4.11}$$

There is very good agreement between this relationship and data taken from the eruption clouds of St. Helens in 1980, Redoubt in 1990 and Pinatubo in 1991 (Sparks *et al.* 1997).

5. Gravity currents: pyroclastic flows, turbidity currents, lava domes

Gravity currents occur whenever fluid of one density flows primarily horizontally into fluid of a different density. They are driven by horizontal pressure gradients which result from the buoyancy (either relatively positive or negative) of the current. (Primarily vertical propagation, driven by vertical pressure gradients due to horizontal buoyancy differences is studied mainly as plumes.) Gravity currents occur in a wide range of natural (and industrial) situations, including: the spread of oil on the surface of water; the motion of a dense, ash-laden pyroclastic flow along the ground (figure 15); the flow of sand- and silt-laden water from the continental shelves across the ocean floor in what is termed a turbidity current; the propagation of relatively dense sea breezes in the atmosphere; and the slow motion of thick, sticky lava in a volcanic crater. A nice review of the field is given by Simpson (1997).

Buoyancy forces are always important in the motion of gravity currents, which can flow either at large Reynolds numbers, when the buoyancy forces are balanced by inertia forces, or at low Reynolds numbers, when the buoyancy forces are balanced by viscous forces (and inertial forces are negligible)†. Each current can be essentially two-dimensional, axisymmetric or influenced by topography. Some gravity currents are the result of a rather rapid release of a given volume of fluid; some are due to a continual flux of fluid; and other possibilities have also been investigated. This section develops fundamental concepts used to describe gravity currents and discusses a few geological applications of recent interest.

5.1. *Compositional, large Reynolds number gravity currents*

The simplest gravity currents are driven by differences in composition, such as salt. The fact that the currents tend to be very much longer than they are thick immediately suggests the use of the shallow water equations (Whitham, 1974) wherein the vertical pressure gradient is hydrostatic. Under this assumption, the equations for conservation of mass and momentum (neglecting any entrainment of the ambient which might occur) for a two-dimensional current of density ρ_c and height $h(x,t)$ propagating with horizontal velocity $u(x,t)$ below a layer of fluid of density $\rho_0 < \rho_c$ (as sketched in figure 16) are

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) = 0 \qquad \text{and} \qquad \frac{\partial}{\partial t}(uh) + \frac{\partial}{\partial x}(u^2 h) + g' h \frac{\partial h}{\partial x} = 0 \; , \qquad (5.1\text{a, b})$$

where the current propagates under horizontal pressure gradient $\partial p/\partial x = -\rho_0 g' \partial h/\partial x$ and reduced gravity $g' = (\rho_c - \rho_0) g/\rho_0$. In order to solve equation (5.1) initial conditions

† Flows at intermediate values of the Reynolds number, where all three forces are roughly balanced, are rarely encountered.

are required – generally describing how the current was initiated – and two boundary conditions – one at each end of the current. The condition at $x = 0$, where in the laboratory there is a vertical wall, is generally $u(0,t) = 0$. The use of Bernoulli's theorem indicates that the velocity at the nose of the current u_N and the depth at the head h_N are related by

$$u_N = Fr(g^{'}h_N)^{1/2} \ , \tag{5.2}$$

where the Froude number Fr is a constant, determined by perfect fluid theory to be $\sqrt{2}$ for an infinitely deep upper layer or by experiments on real fluids to be 1.19†. The difference between these two values represents the effects of turbulent Reynolds stresses and viscous drag in the vicinity of the head in a real fluid, which bring about additional momentum transfer at the head and hence retard the flow. [If the undisturbed ambient layer is of finite height $H_a < h_N/0.075,\ Fr = 0.5(h_N/H_a)^{1/3}$.]

There exists a useful family of similarity solutions to (5.1) and (5.2). For a current of fixed volume A per unit width intruding into a layer of very large depth, the solution can be found by substituting into (5.1) and (5.2) the forms

$$g^{'}h(x,t) = \dot{x}_N^2(t)\mathcal{H}(\sigma) \quad \text{and} \quad u(x,t) = \dot{x}_N(t)\mathcal{U}(\sigma) \ , \tag{5.3a, b}$$

where the similarity variable $\sigma = x/x_N(t)$, to determine that

$$\mathcal{H}(\sigma) = \tfrac{1}{4}(\sigma^2 - 1) + Fr^{-2} \ , \ \ \mathcal{U}(\sigma) = s \quad \text{and} \quad x_N(t) = \mathcal{C}(g'A)^{1/3}t^{2/3} \ , \tag{5.4a, b, c}$$

where $\mathcal{C} = [27Fr^2/(12 - 2Fr^2)]^{1/3}$ and $x_N(t)$ is the length of the current. This solution is valid some time after the release of the current. If the release takes place by instantaneously lifting a lock gate in front of a rectangular region of heavy fluid, as is frequently

† Under quite general conditions in an infinitely deep ambient, u_N can only be a function of g' and h_N. Dimensional arguments then indicate that the only non-dimensional quantity, $u_N/(g'h_N)^{1/2}$, must be a constant (because there is nothing else for it to depend upon).

the case in laboratory experiments, the current starts in a "slumping phase", wherein the initial column collapses, a nose is driven forward at virtually constant speed and a return bore in the upper fluid propagates backwards to conserve volume. Once the bore has reflected off the back wall and caught up with the front of the current the similarity form of solution (5.4) becomes valid.

Mathematically different, but qualitatively similar, similarity solutions exist if either the volume increases as a power law in time due to an input flux at the origin or in an axisymmetric geometry, or both (Bonnecaze *et al.* 1993, 1995).

A less rigorous, but extremely useful, approach is to consider a simple "box" model of the flow, which represents the current as a series of rectangles of equal volume with no horizontal variation of properties within the flow. In two dimensions this requires the solution of

$$x_N h_N = A \quad \text{and} \quad \dot{x}_N(t) = Fr(g' h_N)^{1/2} \ , \tag{5.5a, b}$$

which, with initial condition $x_N(0) = 0$, is identical to (5.4c) except that $\mathcal{C} = (3Fr/2)^{2/3}$. The difference between these two values of $\mathcal{C}$ is no more than 10% for $1.2 < Fr < \sqrt{2}$. In axisymmetric geometry, the radial extent $r_N(t)$ of an instantaneously released, fixed volume of fluid V is given by $r_N(t) = \mathcal{C}_r(g'V)^{1/4}t^{1/2}$, where the box model and similarity solution values of $\mathcal{C}_r$ are $(4Fr^2/\pi)^{1/4}$ and $(4Fr^2/\pi)^{1/4}[4/(4-Fr^2)]^{1/4}$. A good qualitative feel for the solutions can also be obtained from evaluating and then equating the total buoyancy and inertial forces in the current. This approach is described in the Appendix of Huppert (1982).

Entrainment of ambient fluid into the flow has been investigated by experimentally following the intrusion of an alkaline current into an acidic ambient. Entrainment was seen to take place almost entirely at the head of the current due to shear instabilities on the interface between the current and the ambient and by the over-riding of the (relatively less dense) ambient fluid by the head. An entrainment or dilution ratio E, defined as the ratio of the volumes of ambient and original fluid in the head, which hence must be non-negative, can be shown by dimensional analysis, and confirmed by experiment, to be independent of g', and to be given in two dimensions by

$$E = [1 - c_1 y_N / A_S^{\frac{1}{2}}]^{-c_2} - 1 \ , \tag{5.6}$$

where A_S is the volume per unit width of fluid in the head at the end of the slumping phase (which occurs when the current has propagated about ten lock lengths), y_N is the position of the head beyond the slumping point, and $c_1 \approx 0.05$ and $c_2 \approx 1.5$ are empirical constants determined by the roughness of the floor. Note that (5.6) is consistent with the rather surprising result that the entrainment is essentially zero in the slumping phase (when $y_N < 0$).

5.2. *Particle-driven gravity currents*

When heavy (or possibly relatively less dense) particles drive the flow the major new addition is that the particles fall (or rise) out of the flow and the driving buoyancy continually decreases. The approach most frequently taken to analyze the sedimentation if the concentration is not too large is to assume that the (high Reynolds number) flow is sufficiently turbulent to maintain a vertically uniform particle concentration in the main body of the current. However, at the base of the flow, where the fluid velocities diminish appreciably, the settling of particles occurs at the (low Reynolds number) Stokes velocity V_s in otherwise quiescent fluid. Quantitatively, this indicates that, neglecting particle *advection* for the moment and assuming that the particles are all of one size, if N_p (which is possibly a function of time and position) denotes the total number of particles per unit horizontal area in a layer of depth h, the change of N_p in time δt, δN_p, due only to the sedimentation is given by $\delta N_p = -V_s C_0 \delta t$, where C_0 is the (number) concentration (per unit volume) just above the base of the flow. Vigorous turbulent mixing implies that $C_0 = N_p/h$, which (on taking the appropriate infinitesimal limits) indicates that $\dot{N}_p = -V_s N_p/h$, a relationship which has been carefully verified by experiments (Martin & Nokes, 1988). The incorporation of advection of the particles by the mean flow then results in

$$\frac{D\Phi}{Dt} \equiv \frac{\partial \Phi}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} \Phi = -V_s \Phi/h \ , \qquad (5.7\text{a, b})$$

where Φ is the volume concentration of particles.

Shallow water equations, akin to (5.1), and incorporating (5.7), can be derived (Bonnecaze *et al.* 1993,1995). There are no similarity solutions and recourse, in general, has to be made to numerical solution, although it is also possible to develop asymptotic, analytic solutions, based on the smallness of $\beta_s = V_s/(g_0' h_0)^{1/2}$, where g_0' is the *initial* reduced gravity of the system (Harris, Hogg & Huppert 2000).

Insightful, box-model solutions are relatively straightforward to obtain. In two dimensions this requires, for the instantaneous release of a fixed volume A per unit width of particle-laden fluid, the solution of

$$\dot{x}_N = Fr(g_p' \Phi A/x_N)^{1/2} \quad \text{and} \quad \dot{\Phi} = -V_s x_N \Phi/A \ , \qquad (5.8\text{a, b})$$

where $g_p' = (\rho_p - \rho_a)g/\rho_a$, ρ_p is the density of the particles, and the density of the interstitial fluid in the current has been assumed to identical to that of the ambient. Appropriate initial conditions are

$$x_N = 0 \quad \text{and} \quad \Phi = \Phi_0 \qquad (t = 0) \ . \qquad (5.9\text{a, b})$$

Dividing (5.8b) by (5.8a) and integrating the resulting ordinary differential equation subject to (5.9), we obtain

$$\Phi(t) = \left(\Phi_0^{1/2} - \lambda_p x_N^{5/2}\right)^2 \ , \qquad (5.10)$$

where $\lambda_p = 0.2V_s/(Fr^2 g_p^{'} A^3)^{1/2}$, from which we can deduce immediately that the current ceases to flow ($\Phi = 0$) at $x_N = l_\infty \equiv (\Phi_0^{1/2}/\lambda_p)^{2/5}$. Introducing non-dimensional variables $\Phi = \phi/\phi_0$ and $\xi = x_N/l_\infty$, substituting (5.10) into (5.8a) and using (5.9a), we determine that

$$\tau = \int_0^\xi s^{1/2}(1 - s^{5/2})^{-1} ds \equiv \mathcal{F}(\xi) \tag{5.11}$$

in terms of a dimensionless time τ given by $\tau = Fr(g_p^{'} A\Phi_0)^{1/2}(x_N/l_\infty)^{-3/2}\ t$.

In order to evaluate the resulting deposit distribution, we argue that in time δt, a mass per unit width $\delta M = -\rho_p A \delta\Phi$ is deposited uniformly over a length x_N to lead to a deposit density $\delta\eta = -\rho_p A\delta\Phi/x_N$. Thus the total deposit density (of dimensions ML^{-2}) after the flow has ceased is given by

$$\eta = -\rho_p A \int_{x_N}^{l_\infty} z^{-1}\frac{d\Phi}{dz} dz = \frac{25\phi_0\rho_p A}{12 x l_\infty}\left(1 - \frac{8}{5}\xi^{3/2} + \frac{3}{5}\xi^4\right). \tag{5.12a, b}$$

Similar results can be obtained for axisymmetric particle-driven gravity currents. Evaluation of the details are left to the reader as an exercise, with the answers given in Huppert & Dade (1998).

The erosion of a sedimentary bed due to the pick up of particles can play an important role in particle-driven flows. The erosion of a bed by a shear flow, in such a way as to increase the buoyancy, and hence the shear, to lead to a self-accelerating current, a process often called autosuspension, was first considered, independently, by Pantin (1979) and Parker (1982). They derived a fifth order, nonlinear ordinary differential system to describe the evolution of the flow and analyzed the behaviour of the resulting solutions using phase plane techniques. Unfortunately, the values of some of the parameters that appear in the theory are not known and the predicted criteria for particle erosion has not yet been subjected to careful experimental investigation.

An expanded version of this sub-section, which brings out additional concepts and examples can be found in Huppert (1998b).

5.3. *Some geological applications*

There are many geological situations in which particle-driven gravity currents play a fundamental role. One example concerns the motion of pyroclastic flows, resulting from the collapse of volcanic eruption columns. The largest flows have been modelled as isothermal, relatively low concentration entities which spread radially along the ground. A particular problem posed, and answered, by Dade & Huppert (1996) was the determination of the initial conditions of the flow, and especially the initial particulate concentration, given the observed radial distribution of the final deposit. Having set up a general framework, Dade & Huppert applied it specifically to analyzing one of the largest eruptions in the last 10,000 years, the eruption of Taupo on the North Island of New Zealand in AD 186. Approximately $30km^3$ of solid material was distributed in a roughly axisymmetric fashion around a vent up to a radius of 80 km as a result of the eruption. The total volumetric flux, after column collapse and the associated entrainment of air, was found to be of order $40km^3\ s^{-1}$, over a period of approximately 15 minutes. The initial solids concentration in the pyroclastic flow was around 0.3% by volume – a result consistent with the initial assumption of low particle concentrations.

This low concentration, which was greeted with surprise by some Earth scientists, is consistent with the idea, well known to geologists, that some pyroclastic flows can, quite suddenly, lift up into the atmosphere to form what are called co-ignimbrite plumes. These occur because the small, hot ash particles can heat the air in the flow sufficiently that the bulk density of hot air plus particles exceeds that of the relatively cold, particle-free air of the surrounding atmosphere. The general analysis of such particle-driven flows with less dense interstitial fluid can be analyzed using the concepts (box-models, shallow water theory, etc.) of the last subsection (Sparks *et al.* 1993; Hogg, Huppert & Hallworth 1999). A simple but instructive calculation equates the density difference purely due to thermal differences $\Delta\rho_T$ to the density difference due to the particulate concentration $\Delta\rho_c = C\rho_p$, where C is the fractional volume concentration of particles of density ρ_p. The equality of these two contributions to the density would strictly be appropriate just at lift-off, but it allows an approximate estimate of the particle concentration to be obtained. Thus with the densities of air at 1000 and 10°C being 0.28 and 1.3 kg m^{-3} respectively and $\rho_p \sim 2500kg\ m^{-3}$, $C \sim 0.0004$, which indicates that the calculated concentration of particles in the Taupo eruption, although considered low by some geologists, was already considerably larger than that in many other such flows.

Particle-driven flows with less dense interstitial fluid can also occur when sand- and silt-laden fresh water rivers discharge into a (salty) ocean. The particulate concentration can be sufficiently large that the discharge flows along the bottom of the ocean for many kilometres until, when sufficient particles have fallen out of the flow, the interstitial fluid rises and thereby mixes fresh water into the ocean a considerable distance off shore. Such a situation is believed to be permanently operating in some ten of the world's largest rivers, many of them in China, with the most famous example being the Yellow River.

Large suspension-driven flows at the bottom of the oceans, known as turbidity currents, have been well documented by geologists and are the main mechanism by which sediment from land is transported into the deep sea. Volumes as large as $500km^3$ of sand and silt can propagate many hundreds of kilometres across the ocean floor, as was first effectively realized in 1929 when, owing to an earthquake on the Grand Banks off the eastern coast of the USA, submarine cables were broken sequentially by the resulting turbidity current as it propagated across the Atlantic. Sequential turbidite flows can lay down a series of beds which can act as reservoirs for oil. One of the larger turbidite flows to have been continuously traced on the ocean floor is found on the Hatteras Plain off the eastern coast of North America and is known as the 'Black-Shell turbidite' because of the many small black shells that litter the deposit. The turbidite lies in water 5.5 km deep, covers an area of at least 44×10^3 km^2 and extends for more than 500 km along a fairly straight, two-dimensional channel flanked by abyssal hills. Using the observed thickness of the deposit as a function of the distance along the Hatteras Abyssal Channel, a mean fall speed V_s of 0.08 $cm\ s^{-1}$ (corresponding to silt-sized particles with an effective diameter of $32\mu m$) and the box-model results developed in §5.2, Dade & Huppert (1994) showed that the deposit resulted from an initial surge 30 km long, 300 m high and approximately 200 km wide ($\approx 2,000$ km^3) containing particulate matter which made up 5% by volume, and 13% by weight, of the surge.

As described at the end of §4, a volcanic eruption plume which intrudes laterally into the atmosphere at its level of neutral buoyancy, contains many small (heavy) ash particles. Consider the penetration as a steady axisymmetric flow with radial velocity u_r of a layer of turbulent fluid of thickness h. The rate of change of the mass M of suspended particles in the current with radial distance r will be given, following (5.7b), by $dM/dr = -V_s M/(u_r h)$. Because the flow is steady, the flux $Q = 2\pi r h u_r$ is constant and so

$$M = M_0 \exp\Big[-\pi V_s (r^2 - r_0^2)/Q\Big] , \qquad (5.13)$$

where M_0 is the mass of particles in the current at radius r_0. The agreement between the prediction of (5.13) and data from both laboratory experiments and from the measured sedimentation density of, for example, the deposit on the island of San Miguel in the Azores, from the umbrella cloud associated with the eruption of the Agua de Pau Volcano about 3000 BCE, is surprisingly good (Sparks *et al.* 1997).

5.4. *Low Reynolds number gravity currents*

When viscous forces dominate inertial forces, the well-known concepts of lubrication theory (Davis, ch D) can be used to determine the flow. The formulation is then similar to that employed in analyzing parallel flow, as described at the beginning of §3. The paradigm situation (Huppert, 1982) considers the spreading in either a two-dimensional or axisymmetric geometry of a Newtonian fluid above a horizontal base and seeks the shape and resultant rate of spreading of the current under the assumption that its horizontal extent greatly exceeds its thickness. The pressure p within the current is hydrostatic and thus the pressure gradient driving the flow is proportional to the slope of the height of the (unknown) free surface h. Hence for viscous fluid of density ρ and kinematic viscosity ν extruded at a line source in a two-dimensional fashion, as sketched in figure 17, the parabolic, horizontal velocity profile is directly proportional to $\partial h/\partial x$, where x is the horizontal co-ordinate perpendicular to the source (with $x = 0$ at the source). Use of the depth-integrated continuity equation (with details not given here because they are supplied in Huppert, 1982 and 1986) then indicates that $h(x,t)$ satisfies the singular nonlinear diffusion equation

$$\frac{\partial h}{\partial t} - (g/3\nu)\frac{\partial}{\partial x}\left(h^3\frac{\partial h}{\partial x}\right) = 0 \ , \tag{5.14}$$

with the one boundary condition $h[l(t),t] = 0$, where $l(t)$ is the length of the current. If the extrusion of fluid takes place so that the cross-sectional volume per unit width of the current increases with time as qt^α where α is a (non-negative) constant, (5.14) must be solved subject to the global continuity equation

$$\int_0^{l(t)} h(x,t)dx = qt^\alpha \ . \tag{5.15}$$

With difficulty, numerical solutions of (5.14) and (5.15) can be obtained for any given initial conditions. Alternatively, similarity solutions, to which all solutions with sufficiently regular initial conditions will tend, except possibly at $x = 0$, are easy to obtain. Indeed, determination of the independent similarity variable is sufficient to show that

$$l(t) = \eta_N(\alpha)(gq^3/3\nu)^{1/5}t^{(3\alpha+1)/5} , \tag{5.16}$$

where the constant $\eta_N(\alpha)$ can be calculated to be a monotonically decreasing function of α with $1.41 > \eta_N > 0.85$ for $0 < \alpha < 2.5$. (Equation (5.16) can also be obtained by globally equating viscous forces to buoyancy forces.)

For an axisymmetric current of radius $r_N(t)$, extruded from the origin so that the volume of the current is Qt^α, the argument outlined above leads to the result

$$r_N(t) = \xi_N(\alpha)(gQ^3/3\nu)^{1/5}t^{(3\alpha+1)/8} , \tag{5.17}$$

where $\xi_N(\alpha)$ is also a monotonically decreasing function with $0.90 > \xi_N > 0.65$ for $0 < \alpha < 2.5$. In particular, for a viscous current of constant volume, $r_N(t) \propto t^{1/8}$. Numerous laboratory experiments confirm the accuracy of the theoretical approach and predictions.

One of the motivations of the original analysis was to apply the results to examine and interpret the spreading of lava domes, which are often extruded in volcanic craters. For example, commencing in late April 1979, a lava dome spread across the relatively flat base of the 1200 m high Soufriere of St. Vincent in the West Indies, following a series of explosive eruptions starting on 13 April. The lava dome slowly increased in volume, and spread horizontally, for the next five months, by which time the pancake-shaped dome had a height of 130 m, a mean diameter of 870 m and a volume of 50 x $10^6 m^3$. Careful measurements during this time indicated that the volume $V(t)$ and radius $r(t)$ as functions of time were well represented by $V(t) = 930t^{0.66}$ and $r(t) = 90t^{0.39}$ (in SI units).

The theoretical result (5.17) can then be used in two different ways. Firstly, the power-law representation of the measured volume indicates that for this dome $\alpha = 0.66$, which should lead to a radial time dependence of $(3\alpha + 1)/8 = 0.37$, in good agreement with the measured value of 0.39. Secondly, the observed coefficient in the radial spread of 90 can be substituted into (5.17) to determine a value of $6 \times 10^7\ m^2 s^{-1}$ for ν.

An isothermal Newtonian fluid captures some of the features of some spreading lava domes, and in so doing acts as the foundation for more developed models which incorporate extra effects. Amongst the effects that need to be included are the cooling of the lava as it flows (possibly in a non-Newtonian way) to form a resistive crust on the surface. The field has advanced mainly due to extensive laboratory experiments, with the results backed up by scaling arguments – detailed solutions to governing equations are still awaited. One aim of the research has been to describe the different flow morphologies observed in recently active domes, which include, aside from the Soufriere of St. Vincent, Unzen, Merapi and others.

Fink and Griffiths (1990) conducted a series of experiments in which liquid polyethylene glycol (PEG) was extruded at a constant rate from a point source to flow along a horizontal boundary below a layer of effectively cold water. The water temperature was maintained below the freezing temperature of the PEG. For those experiments with the warmest layer temperatures, the PEG did not solidify and spread as calculated above for an isothermal, constant viscosity (Newtonian) fluid. In experiments with cooler layers, a solid PEG crust formed and one of four different morphologies developed. These were categorized by Fink and Griffiths as: levees; folds; rifts; and pillows. The transition between the different morphologies (and from the formation of no crust at all) was parameterized in terms of a single variable Ψ, which is the ratio of the time for the initially warm PEG to solidify to an advection time scale.

Fink and Griffiths (1998) extended these experiments to mimic a (plastic) yield strength in magma by adding kaolin powder to the liquid PEG. The existence of the resulting internal yield stress had a strong influence on the behaviour of the flows, which displayed morphologies quite different from the previous flows. They argued that the latter experiments are better analogues of the more viscous magma domes and, in particular, the numerous lava domes recently observed on Venus.

6. Extra Topics

There remain numerous areas of geological fluid mechanics which have not been described here. Many (but not all) of them have as a common feature the flow of fluid through a porous medium. The Navier-Stokes momentum equation is then replaced by Darcy's law (or some extension of it), whereby the fluid transport velocity is directly proportional to the driving pressure gradient, with the constant of proportionality being the permeability of the medium divided by the dynamic viscosity of the fluid (Phillips, 1991). A large number of interestingly different flows can result.

Deep-sea vents and the hydrothermal circulation. One of the most exciting recent events in the Earth Sciences has been the discovery by small, manned submersibles of the existence of a chain of vents littering the bottom of the sea, centred on the mid-ocean ridges. As depicted in figure 18, hot water gushes through holes $\sim 70cm$ in diameter, at velocities of $\sim 5m\ s^{-1}$, into an ocean whose ambient temperature (at that depth) is quite steady at approximately 2°C. These turbulent, entraining plumes, which are part of the hydrothermal circulation, are driven by the heat lost from the hot magma chambers several kilometres beneath the ridge axes. Relatively cold sea water penetrates the oceanic crust over a broad area several tens of kilometres on either side of the ridge axis. As water flows down through the crust it is heated to temperatures as high as 400°C without vaporising, because of the high pressures. At such temperatures the density of the water is ~ 0.7 times that of cold sea water and hence extremely buoyant. The venting at individual sources represents the return flow. Calculations of thermal flow in a porous medium driven by a heat source, in some ways similar to the high Rayleigh number calculations in Chapter X.y by Linden (but see also Phillips, §7.6) reflect many of the observed features of the flow. There are indications, however, that the crust is heavily fractured and that the flow is strongly focused along numerous cracks, which will require augmentation of the governing equations.

The turbulent discharge of this less dense, hot water at the bottom of the ocean is another example (following §4.2) of a buoyant plume as described in Chapter X.y (Linden). Chemical analyses indicate that the mixing of the entrained sea water into the hydrothermal fluid of the plume forms a black precipitate, which has led to the plumes being called "black smokers". (A very small number of plumes precipitate white particles and are hence known as white smokers.) Some of the heavy particles fall out of the plume and calculations indicate that more than a half are then re-entrained back into the plume. After a rise of a few hundred metres, the plumes reach their level of neutral buoyancy, as quantitatively predicted by (4.7), and flow out horizontally into the ocean.

Aside from the chemical and physical importance (and their great fluid mechanical interest), black smokers have introduced new concepts into biology. At that depth in the ocean, sunlight is totally absent and chemosynthesis replaces photosynthesis as the mechanism for life. Completely new biological organisms have been found around the vents where they thrive on the hot, chemically rich environment. It has been seriously suggested that they will represent a significant food source in the future.

Flow and reactions in porous sedimentary rock. A foundation for those aspects of flow in a porous medium that are relevant to sedimentary geology is clearly described by Phillips (1991) with a stimulating summary given in Phillips (1990). One of the central themes of the book and paper is the analysis of slow flows that react thermally and chemically with the solid matrix through which they percolate (c.f. the flow through a mushy layer described by Worster, chapter X.y). The controlling processes in such flows are the fluid mechanical ones of advection and diffusion, which specify the rate at which reactants can be supplied to the reaction sites, and the chemical ones which specify the rates of reaction. Phillips categorizes three different types of flow. First, isothermal reaction fronts, which propagate like broad shock waves altering the chemical composition (of both the fluid and the enveloping matrix) from a constant, specified value ahead of the travelling front to a different constant value behind it. These can occur, for example, as a result of the reaction which turns limestone or calcite into dolomite, or, maybe more generally, when an acidic aqueous solution dissolves material in the solid rock matrix.

With $c(\mathbf{r}, t)$ and $\mathbf{u}(\mathbf{r}, t)$ denoting the concentration of the reactant (e.g magnesium ions, acid, ...) and transport velocity in the fluid as functions of space and time, the governing conservation equation becomes

$$\phi \frac{\partial c}{\partial t} + \nabla \cdot (\mathbf{u}c) = \phi D_e \nabla^2 c + \phi Q_c \ , \tag{6.1}$$

where ϕ is the porosity (fluid volume fraction), D_e the effective diffusivity of concentration and the source term Q_c may be a function of the concentration of each of the reactants, temperature, pressure, space, time and possibly other variables. Note that ϕ (typically $\ll 1$) appears as a pre-multiplicative factor in front of those terms that reflect that c (the acid concentration, say) is confined to the fluid. As a consequence, as seen on dividing (6.1) by ϕ, variations in the acid concentration are advected with fluid velocity $\mathbf{q}/\phi$, which is much larger than the transport velocity $\mathbf{q}$. With $m(\mathbf{r}, t)$ denoting the concentration of the other reactant in the solid (e.g. $CaCO_3$; let us call it mineral), the governing equation becomes

$$(1-\phi)\frac{\partial m}{\partial t} = -\gamma_c(1-\phi)Q_c \ , \tag{6.2}$$

where γ_c is a (stoichiometric) constant dependent of the particular chemistry of the reaction under investigation.

With the simplest assumption, that $Q_c \propto mc$, Phillips, Hinch & Bhatt (1990) (and others) treat ϕ as a constant and develop analytical one-dimensional travelling wave solutions of the form

$$c = F(\zeta = x - Vt) \qquad m = G(\zeta) \tag{6.3a, b}$$

with $F \to 0$ (all the acid is used), $G \to m_0$ (the initial mineral concentration) as $\zeta \to \infty$ and $F \to c_0$ (the initial acid concentration), $G \to 0$ (mineralisation is complete) as $\zeta \to -\infty$. The resultant velocity with which the front moves, V, is typically a small fraction of the velocity of the acid.

Hinch & Bhatt go on to investigate the linear stability of the front by allowing the permeability (but not the porosity) to be a function of m. This is a general Saffman-Taylor situation (Couder, §X.y) and hence if the permeability decreases with m (i.e. in the direction of motion) the front is unstable; otherwise it is stable.

In other situations there are initial spatial gradients of m. A simple travelling wave representation is then not possible because the reaction can continue at different rates within the matrix. Phillips terms this a gradient reaction in which the rate of reaction or deposition in the pores is proportional to the gradient of temperature and to the interstitial fluid velocity in the direction of the gradient. In a faulted or fractured medium, the fluid velocities in the fracture network are typically very much larger than those in the matrix, and the rates of mineral deposition are correspondingly larger, leading to the characteristically vinuous structure of mineral deposits. Finally, Phillips defines mixing zone reactions which occur as a result of intimate mixing between different fluids as they seep through a porous matrix, such as can occur when sea water infiltrates a coastal aquifer initially saturated with fresh water. The ability of areas of relatively high permeability to attract and focus flow from low permeability regions draws streamlines closer together and hence promotes mixing.

Geothermal reservoirs and their replenishment. Another exciting area of flow in porous media concerns advection and convection in geothermal reservoirs made up of fractured rock at temperatures of up to 400°C which are located in the upper 10 km of the Earth's crust. A number of these on land have been used over the last thirty years or so as thermal energy sources on a commercial scale, although the use of naturally heated water has been common in spas and even in some forms of central heating for a few millennia. One fascinating problem, associated with such areas is the formation of geysers, such as at Yellowstone National Park in the United States, which can erupt with remarkable regularity although the essential mechanisms are still not understood. Another problem concerns the input of cold water to replenish warm water extracted by commercial hydrothermal heating plants.

A simple situation, which nevertheless illustrates many of the essential features, occurs when cold water at temperature T_i is input at a small source with flux Q (of dimensions L^2T^{-1}) in a saturated, hot, two-dimensional porous medium at temperature $T_\infty (> T_i)$. The thermal balance for the temperature T, which is identical in the solid and adjoining fluid, can be written as

$$\overline{\rho c_p}\frac{\partial T}{\partial t} + (\rho c_p)_l (Q/2\pi r)\frac{\partial T}{\partial r} = \overline{\rho c_p \kappa} r^{-1}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) , \tag{6.4}$$

Woods (1999), where r is the radial co-ordinate and an overbar indicates a mean value for the system weighted with respect to the volume occupied, i.e. for any variable σ say, $\overline{\sigma} = \phi\sigma_l + (1-\phi)\sigma_s$, where σ_l and σ_s are the values of σ in the liquid and solid (matrix) respectively (Worster X.y). The different coefficients in (6.4) reflect the fact that heat diffuses throughout the matrix but is advected only by the fluid motion through the pores.

Because there is no externally imposed length scale there is a similarity solution to (6.4) in terms of $\eta = r/2(\kappa_m t)^{1/2}$, where $\kappa_m = \overline{\rho c_p \kappa}/\overline{\rho c_p}$, and the one non-dimensional parameter $\lambda = [(\rho c_p)_l/\overline{\rho c_p}](Q/\kappa_m) \sim (Q/\kappa_m)$, given by

$$T(\eta) = T_i + (T_\infty - T_i)P(\frac{1}{2}\lambda, \eta^2) \ , \tag{6.5}$$

where $P(a, x)$ is the incomplete gamma function. For large values of λ, which correspond to input fluxes much larger than thermal diffusion, there is negligible conduction of heat from the far field. Instead the input liquid is heated to T_∞ by extraction of heat from the rock matrix near the source. The temperature jumps quite rapidly from T_i to T_∞ over a thin region at a radius [given by $\eta = (\lambda/2)^{1/2} \equiv \eta_T$] considerably smaller than that to which the input liquid has penetrated [given by $\eta = \eta_T/(\pi\phi)^{1/2}$]. For $\lambda \ll 1$ the liquid front advances more slowly than the rate at which heat is diffused and the problem approaches the purely thermal diffusion situation discussed by Carslaw and Jaeger (1980). For almost all real geothermal systems $\lambda \gg 1$.

If the porous rock is at a temperature above the boiling point of the input liquid, vapour may be produced ahead of the advancing front. The motion may then be determined by combining: Darcy's law for the flow of vapour in the pore spaces; the perfect gas law; and the mass conservation equation; in the form

$$\mathbf{u} = (-k/\mu_v)\nabla P; \qquad p = \rho_v RT; \qquad \text{and} \qquad \phi\frac{\partial \rho_v}{\partial t} + \nabla \cdot (\rho_v \mathbf{u}), = 0 \ , \tag{6.6a, b.c}$$

where subscript v denotes the vapour phase. This leads to the nonlinear diffusion equation

$$\frac{\partial}{\partial t}(p/T) = \Gamma\nabla \cdot (p\nabla p/T) \ , \tag{6.7}$$

where $\Gamma = k/(\phi\mu_v)$ and k is the permeability of the porous medium. Again, on the assumption that the flow is radially symmetric, a similarity solution is obtainable, which has a different structure for small, intermediate and large values of Γ. In practice such boiling fronts can become unstable to yet another manifestation of a Saffman-Taylor instability.

The evolution of the inner core. The largest fluid region of the Earth is the liquid outer core, in which there are vigorous, convectively-driven motions which maintain the all-important magnetic field of the Earth as described by Moffatt (Chapter X.y). The motions are driven by the slow cooling of the Earth, which causes the iron-rich liquid, with minor constituents of between 5 and 15% of sulphur, oxygen, nitrogen and nickel, to solidify almost pure iron on the boundary of the inner core, as mentioned in §1.2. The convection is partially thermal, driven by the heat transfer associated with the rise of warm fluid (and sinking of cold fluid) within the outer core and partially compositional, driven by the release of relatively less dense fluid due to the incorporation of the heavy iron component into the inner core.

A number of (rather complicated) numerical models have been introduced to study this evolution and to evaluate the radius of the inner core $r_c(t)$ as a function of time. A simplifying step was taken by Buffett *et al.* (1996) who developed a new theoretical model for which the results could be obtained analytically, thus allowing a general understanding to be developed of the role of the various parameters, whose explicit values are not very well known. Buffett *et al.* based their model on global heat conservation and the realization that the cooling of the core is regulated by the heat flux $f_m(t)$ that is taken away from the core-mantle boundary by the motions in the sluggish, more massive mantle. They considered the convection in the outer core to be sufficiently vigorous that the fluid is well mixed there, with a uniform potential temperature (the temperature after subtracting off the adiabatic variation) of $T(t)$, which decreases slowly with time. This temperature is equal to the solidification temperature $T_s[r_c(t)]$ of the inner core, which is a strong function of the pressure at $r = r_c$. The thermal evolution of the (radially symmetric) solid inner core can then be determined by solving a thermal diffusion equation in terms of the imposed temperature T_s at the boundary to yield the resultant heat flux at the boundary between the inner and outer core. From this calculation they determined a (fifth order) polynomial for $r_c(t)$, with an approximate solution

$$r_c(t) = r_b \left\{ \int_{t_o}^{t} f_m(t^{'}) dt^{'} / \mathcal{M} \right\}^{1/2} , \tag{6.8}$$

where r_b ($= 3454km$) is the radius of the outer core, t_o is the initiation time of the growth of the inner core ($\sim\ 2 \times 10^9$ years ago), all other variables are grouped in $\mathcal{M} = (2\pi/9) r_b^3 c_p G \rho^2 (\partial T_s / \partial \rho)$ where G is the gravitational constant. Buffett *et al.* (1996) verified this result by a full numerical computation and investigated the relative importance of thermally- and compositionally-driven convection. They concluded that in the early Earth, when the inner core was much smaller than it is today, thermal convection dominated. Their calculations, extended by Lister & Buffett (1995), indicate that at present the contribution made by compositional convection to the ohmic dissipation, and hence the relative amount of energy available to drive the geodynamo (Moffatt, Chapter ??), represents approximately three-quarters of the total.

Compaction. The initial production of melt from a solid, in infinitesimal amounts, occurs at the boundaries between individual grains. When this happens in the Earth, some of the melt migrates away from the matrix from which it was formed and, eventually, finds itself, as part of a much greater volume, flowing into magma chambers, up dykes and partaking in a volcanic eruption. The investigation of the mechanisms by which small amounts of melt percolate through a solid matrix has generated enormous interest over the last two decades amongst Earth scientists, applied mathematicians and fluid dynamicists (as well as being of considerable relevance to areas of metallurgy, petroleum engineering and soil science).

In order for the melt to migrate, the matrix must deform, so as to conserve volume. The resulting motion can be analyzed using the concept that the matrix be described as a high viscosity, compressible fluid and the dynamics of melt and matrix be considered separately. Each is assumed to obey the standard conservation laws, with the interaction between the two fluids coupling their motion. This approach was pioneered by McKenzie (1984), who derives carefully the following *compaction equations*, which have now become standard.

In terms of the densities of fluid and solid, ρ_f and ρ_s, and porosity $\phi(\mathbf{r}, t)$, the mass conservation equations for the two phases are

$$\frac{\partial}{\partial t}(\rho_f \phi) + \nabla \cdot (\rho_f \phi \mathbf{v}) = \mathcal{Q} = -\frac{\partial}{\partial t}[\rho_s(1-\phi)] - \nabla \cdot [\rho_s(1-\phi)\mathbf{V}] \ , \tag{6.9a, b}$$

where $\mathbf{v}$ and $\mathbf{V}$ are the velocities of melt and matrix, and $\mathcal{Q}$ the mass transfer rate, or melting rate, from matrix to melt. The melt, being much less viscous than the matrix, is transported by the matrix deformation as well as flowing relative to it, as described by Darcy's law in the form

$$\phi(\mathbf{v} - \mathbf{V}) = -[a^2 K(\phi)/\mu_f]\nabla(p - \rho_f g z) \ , \tag{6.10}$$

where a is an average grain size and the dimensionless permeability $K(\phi)$ depends on the geometry of the porous network. The corresponding low Reynolds number momentum conservation equation for the (fluid) matrix is written as

$$0 = \nabla(p - \rho_f g z) - (1-\phi)(\rho_s - \rho_f)\mathbf{g} + \nabla \cdot \left\{ \eta \left[\nabla \mathbf{V} + (\nabla \mathbf{V})^T \right] + (\zeta - \frac{2}{3}\eta)(\nabla \cdot \mathbf{V}) \right\} \ , \tag{6.11}$$

where ζ and η, the bulk and shear viscosities of the matrix, may also be functions of ϕ.

Equations (6.9)-(6.11) display a rich variety of solutions. An important length scale which arises from the equations, as first determined by McKenzie on analyzing the simplest problem of the behaviour of a layer with constant porosity above an impermeable boundary, is the *compaction length* $\delta_c = [(\zeta + \frac{4}{3}\eta)K/\mu_f]^{1/2} a$ (typically between 0.1 and 1 km for geological situations), which is independent of the all important density contrast, $\rho_f - \rho_s$ between melt and matrix, although this becomes a controlling factor in the relative velocity and (hence time scale) of the resultant motions. If the porosity decreases with height, finite amplitude solitary waves can be initiated which propagate upwards at velocities dependent on their amplitude. This behaviour has been reproduced in laboratory experiments by Scott, Stevenson & Whitehead (1986) in which buoyant, relatively inviscid water was released at the base of a layer of much more viscous and deformable glycerine. In general, the waves can, dependent on the parameters, have phase velocities greater or less than the fluid velocities and group velocities that are directed either parallel or anti-parallel to gravity. The compaction equations have been used not only to examine the physics of melt extraction from the mantle, but also to study the associated chemical signals. Some of this work is reviewed in the papers which appear in Cann, Elderfield & Laughton (1997).

There are many more topics of immediate concern in understanding the Earth where fluid mechanics plays a central role. These inclued at least such topics as the propagation of plumes through the mantle (Jackson, 1998), mountain building and deformation of continents (England & Jackson, 1989), deposition and evolution of ore deposits (Phillips, 1991) and the role of convection in maintaining the motion of the continental plates (Peltier, 1989). At the change of the millennium we are just beginning to discover the main processes involved in these fluid motions. A combination of penetrating physical reasoning, powerful applied mathematics and imaginative laboratory experiments will be required to reveal the full range of the further fundamental geophysical mechanics that control the evolution and behaviour of our planet.

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CAPTIONS

Figure 1. A diagrammatic Earth showing the solid inner core, the liquid outer core, the mantle, the lithosphere and the covering clouds. Also depicted are deep mantle plumes and erupting volcanoes.

Figure 2. The nondimensional temperature as a function of position for various values of $\kappa t/a^2$ as a result of the temperature of a layer of thickness 2a being initially raised to a temperature excess $T_+ - T_1$ above the ambient.

Figure 3. a) The resulting temperature profile $\theta(z,t)$ when a constant heat flux H is incident on the boundary of a solid with melting temperature T_M. b) The melting of a solid roof due to a thermally convecting fluid beneath it. The released melt is of greater density and is miscible with the fluid.

Figure 4. The melting of a solid roof due to a thermally convecting fluid beneath it. The released melt is of smaller density than the fluid beneath it. a) The Rayleigh number of the melt layer Ra is sufficiently small that heat is transferred through the layer by conduction. b) Ra is so large that the heat is transferred by vigorous convection.

Figure 5. The bulk density of a melt as a function of crystal content X for various values of the total fractional water content by weight N at a pressure of 1.5 k bar, which corresponds to a depth within the Earth of about 500 m (taken from Huppert *et al.* 1982).

Figure 6. a) A fissure eruption in Hawaii. b) The remains of a long fissure in the four corners region of the United States.

Figure 7. The time t_b for a two dimensional fissure to solidify, or the minimum width W_m it attains, as a function of its initial width for a dyke of length H intruding magma at initial temperature 1200° C into country rock at 100° C with a melting temperature of 1150° C (taken from Bruce and Huppert 1989).

Figure 8. The (half-) profile of a crack propagating *vertically* under the influence of buoyancy and elasticity for $\Lambda^* = 0$ and 1 (taken from Lister & Kerr 1991).

Figure 9. The velocity, pressure and void fraction as a function of depth due to an eruption of magma at 1000K with three different water contents in a cylindrical conduit of radius 20 m and length 3 km. Note the fragmentation and dramatic change of behaviour at a void fraction of 75% (courtesy of A. W. Woods).

Figure 10. The decompression phase wherein a flow of pressure p_e and density ρ_e exits a conduit at the local sonic speed a_s and decompresses to the local atmospheric pressure p_0, to obtain density ρ_0 and velocity u_0. The temperature during the decompression phase remains essentially constant.

Figure 11. The velocity and density deficiency at the base of the column in an eruption column with initial velocities $u_0 = 50, 75$ and $200 m\ s^{-1}$. Initial values of the mass flux, temperature and water content are $10^{-1}\ kg\ s^{-1}, 1000K$ and 3% respectively.

Figure 12. The separation in the plane of eruption velocity against mass eruption rate between a Plinian column and a collapsed fountain.

Figure 13. The curve is the theoretical relationship for the height of an eruption column H as a function of the volume eruption rate for standard atmospheric parameter values. The data are from observations of 12 volcanic eruptions in the twentieth century.

Figure 14. a) The umbrella cloud resulting from the eruption of Mt. Redoubt, Alaska in 1991. b) A simulatory laboratory experiment in which a plume of fresh water laden with particles is released into a salinity gradient.

Figure 15. A hot dense pyroclastic flow from the eruption of Mt. Unzen, Japan in 1991.

Figure 16. A sketch of a gravity current initiated by the instantaneous release of a fixed volume of fluid behind a lock gate propagating at high Reynolds number.

Figure 17. A sketch of a viscous (low Reynolds number) gravity current propagating over a horizontal surface.

Figure 18. A typical black smoker vent.