SOLIDIFICATION AND CONVECTION IN THE CORE OF THE EARTH

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The Earth can be divided into three regions: a solid inner core, whose current radius is 1221 km; a liquid outer core, which extends to 3480 km; and a generally solid mantle. Concomitant with heat being lost from all these regions of the earth, the fluid outer core gradually solidifies to extend the radius of the inner core which is made up of almost pure iron. The residual fluid released by the solidification is poor in iron and is thereby less dense than the original melt. This compositional difference, along with thermal differences across the boundary between the core and the mantle, drives vigorous convection in the liquid outer core at a Rayleigh number which is extremely large, and may even exceed 10^{25}. The motion in the outer core is influenced by the rotation of the earth and, because the fluid in the outer core is a good conductor, it is also influenced by electromagnetic effects. Indeed, it is generally believed that the all-important external magnetic field of the earth is maintained by the interaction of the convection within the rotating outer core and the magnetic fields which permeate it. As described, this is a very complicated, highly nonlinear coupled system, concerning which there have been a vast number of intensive studies over many decades (see, for example, Gubbins 1991 and references therein).

The new approach to the solidification problem, which is described in this contribution, is based on global heat conservation and quantifies the dominant features of the thermal evolution of the core without requiring a detailed knowledge of the convective processes involved. Our model, which is described in more detail by Buffett, Huppert, Lister & Woods (1992), is constructed using the idea that the convection in the outer core is sufficiently strong to keep the fluid there well mixed. Because of the slow growth of the inner core, its surface will be in thermodynamic equilibrium with the (spatially uniform) temperature, say T, of the much more voluminous outer core. This temperature, which decreases steadily with time, will be identical to the (prescribed) solidification temperature of the inner core, which is a function of the pressure at the inner core boundary and hence of the radius, say r_i, of the inner core.

The heat extracted from the cooling outer core is transported through the overlying mantle to the surface of the Earth by thermal convection. The relatively sluggish and more massive mantle controls the cooling of the core by regulating the heat flux, say $f_m(t)$, across the core-mantle boundary. While the magnitude and time dependence of $f_m(t)$ will

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depend on the details of mantle convection, in order to focus on the dominant processes involved, we simply prescribe $f_m(t)$ here and investigate the consequent thermal evolution of the core. In spherical geometry, conservation of heat may be expressed as

$$\frac{4\pi}{3}(r_0^3 - r_i^3)c\frac{dT}{dt} - 4\pi r_i^2(L+B)\frac{dr_i}{dt} = 4\pi r_i^2 f_i(t) - 4\pi r_o^2 f_m(t), \tag{1}$$

where c and L are the assumed constant specific heat and latent heat per unit volume, r_0 is the constant radius of the fluid outer core and B, which is due to the change in gravitational energy that results from the rearrangement of mass within the core, can be approximated as (Loper, 1984)

$$B = \Delta\rho g(r_o)r_o\left[\frac{3}{10} - \frac{1}{2}\left(\frac{r_i}{r_o}\right)^2\right], \tag{2}$$

where $\Delta\rho$ is the compositional density jump across the inner core boundary and g is the local acceleration due to gravity.

The conductive heat flux f_i from the inner core, which is given by Fourier's law, is determined from the solution of the thermal diffusion equation in the inner core. Two limiting cases immediately present themselves. These correspond physically to the limits of a perfectly insulating and a perfectly conducting inner core. The difference between these two extremes can be shown to be of order $(r_i/r_o)^3$, which is currently negligible. We hence restrict attention to the simpler case of a perfectly insulating inner core, for which no heat is conducted from the inner core, and $f_i \equiv 0$. A first integral of (1) can then be written as

$$c\int_o^{r_i(t)} \frac{dT}{dr}(r_o^3 - r^3)dr - (L+B)r_i^3 = -3r_0^2\int_0^t f_m(t)dt, \tag{3}$$

where $t = 0$ is the time at which the solid inner core first begins to form.

Over the range of pressures in the core, the solidification temperature is a function of pressure which, by using the hydrostatic equation and Newton's law of gravitation, can be shown to be well approximated by a quadratic function of the radius from the centre of the Earth. Thus we can write

$$T(r) = T(0) - Ar^2, \tag{4}$$

where A is a (fairly well) known constant. Inserting (4) into (3) and carrying out the resulting integrations, we find that

$$\eta^2 + S\eta^3 + B\eta^3(1-\eta^2) = D\int_0^t f_m(t)dt, \tag{5}$$

where

$$\eta = r_i/r_o, \quad D = 3/(Acr_o^3) \tag{6a,b}$$

and the Stefan number

$$S = \frac{1}{3}DLr_o. \tag{7}$$

Using currently accepted values of physical parameters of the Earth, we find that $S \sim 0.4$ and $B \sim 0.6$. Thus since $\eta \sim \frac{1}{3}$ the second and third terms in (5) are significantly smaller than the first. This indicates that at the moment the growth of the inner core is almost entirely controlled by a balance between the cooling of the outer core and the heat flux into the base of the mantle; the latent heat released from the solidification of the inner core, the heat transfer from the inner core and the gravitational energy release play secondary roles. Thus to lowest order, the radius of the inner core can be well represented by

$$r_i(t) = r_o\left\{D\int_0^t f_m(t)dt\right\}^{\frac{1}{2}}. \tag{8}$$

Figure 1 presents the nondimensional radius of the inner core as a function of time, as determined both from the full relationship (5) and from the analogous relationship obtained by assuming that the inner core is a perfect conductor (in which case its temperature would be spatially uniform). At the present time, marked with a $*$, there is very little difference between the two curves. Not until the final stages of the solidification, in approximately another 10^{10} years, will there be a significant difference.

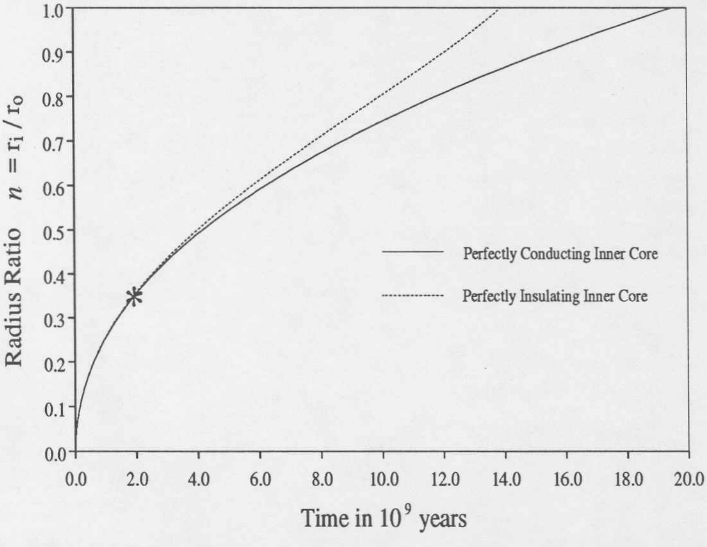

Figure 1: The ratio of the radius of the solid inner core to the outer core, η, as a function of the time since the initiation of the solid inner core. The current values are denoted by a $*$.

References

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